Fixed-motion shadowing on gravitational n-body simulations

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Outline

1. Introduction of Shadowing
2. Refinement Algorithm
3. Fixed-motion Shadowing
4. Results and Discussions

2-galaxy collision by John Dubinski

See the animation …

Introduction

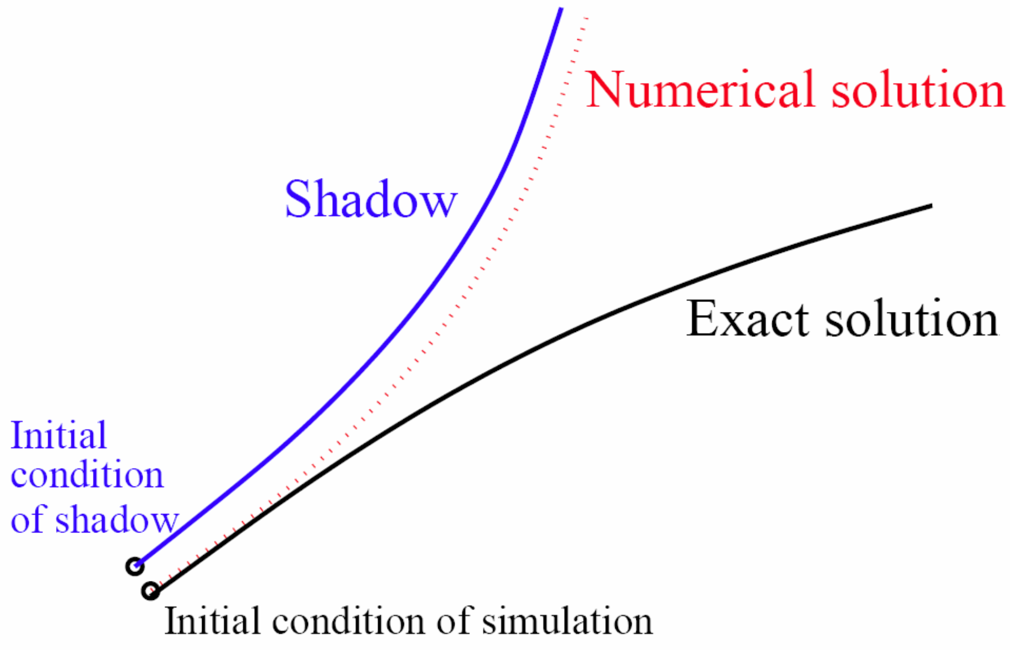
Gravitational N-body problem: $$\begin{pmatrix} \vec{p} \\ \vec{v} \end{pmatrix}' = \begin{pmatrix} \vec{v} \\ \vec{a} \end{pmatrix}$$

where: $$\vec{a}_{ij} = \frac{G m_i m_j}{\left\| \vec{p}_j - \vec{p}_i \right\|^3} \left(\vec{p}_j - \vec{p}_i \right)$$

Softened: $$\vec{a}_{ij} = \frac{G m_i m_j}{\left(\left\| \vec{p}_j - \vec{p}_i \right\|^2 + \varepsilon^2 \right)^{\frac{3}{2}}} \left(\vec{p}_j - \vec{p}_i \right)$$

Introduction

A Shadow: an exact trajectory that stays close to the numerical one for a long time.



Refinement

- Why shadowing?
  - Can we trust numerical simulations?
- GHYS/QT/H refinement algorithm
  - GHYS: Newton-like, noise reduction
  - QT: generalize GHYS to high-dimensional systems
  - H: optimize GHYS/QT to be about 100 times faster
  - Not rigorous shadowing
  - QT believe, failure of refinement means no real solution nearby
  - $O(N^3)$ time

Refinement

Numerical: $y_1, y_2, y_3, \dots y_i, \dots$

Shadow (if exists): $x_1, x_2, x_3, \dots x_i, \dots,$ with $x_{i+1} = \varphi(x_i)$

Correction: $c_i = x_i - y_i$

Error: $y_{i+1} = \widehat{\varphi}(y_i) + e_{i+1}$

So, $c_{i+1} = x_{i+1} - y_{i+1}$

$$= \varphi(x_i) - \varphi(y_i) - e_{i+1}$$

$$= D\varphi(y_i) \cdot c_i - e_{i+1}$$

Fixed-motion Shadowing

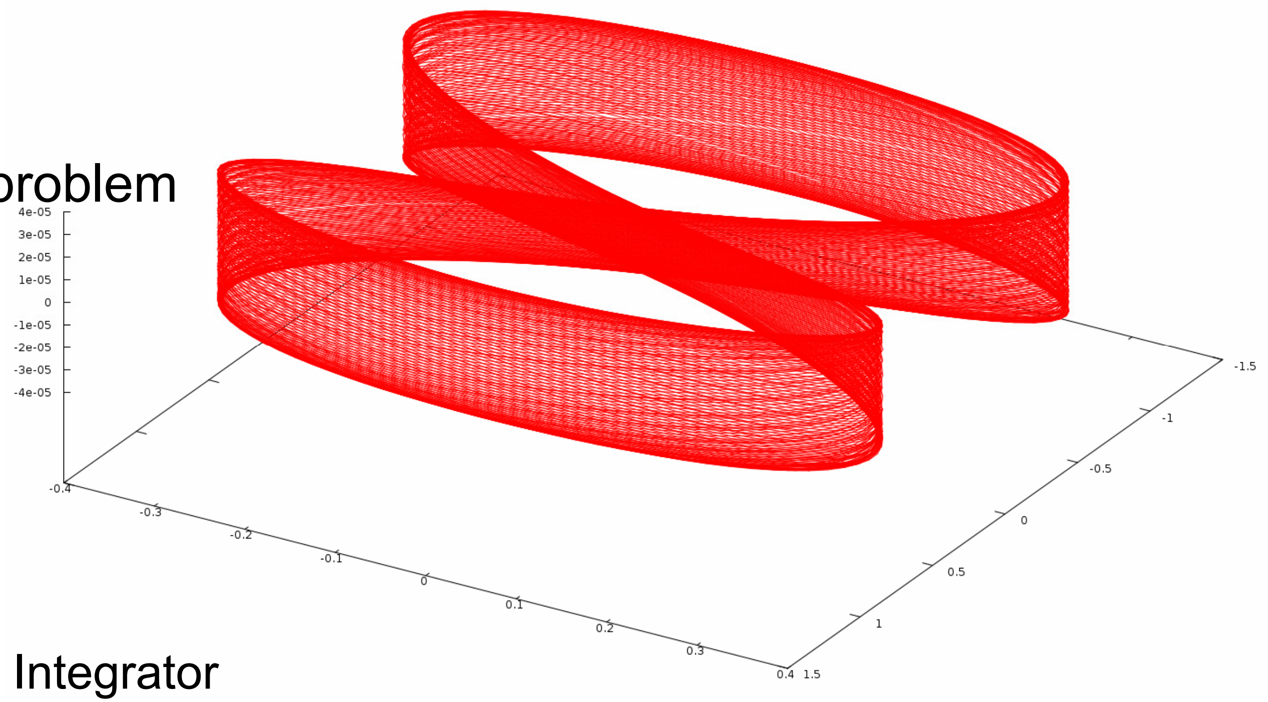
- High dimensional, gravitational n-body simulation
  - John Dubinski, parallel tree-code, leapfrog integrator
  - $N = 10^6$ particles, softened
  - 2-galaxy collision
- Fixed-position shadowing (FPS) : Pinball Machine
  - N particles: $N - M$ fixed, M movable
  - Shadowing M movable particles
  - If $N = 10^6$, only $M << N$ is feasible
- Fixed-motion shadowing (FMS)
  - N particles, all movable
  - Shadowing one or a few particles
  - Interpolation

Fixed-motion Shadowing

- Procedure of FMS
  - Run cheap leapfrog integrator, and approximate force calculation
  - Obtain the noisy trajectory
    - N : number of particles
    - ε : softening
    - h : integrating time step
    - θ : opening angle
  - Separate the noisy trajectory of a particle from the whole trajectory
  - Run GHYS/QT/H refinement algorithm
    - Input : the above trajectory of the particle we want to shadow
    - Output : the longest shadow
    - Plug-in : Hermite interpolation between two time steps, and a new heuristic to accelerate refinement

Fixed-motion Shadowing

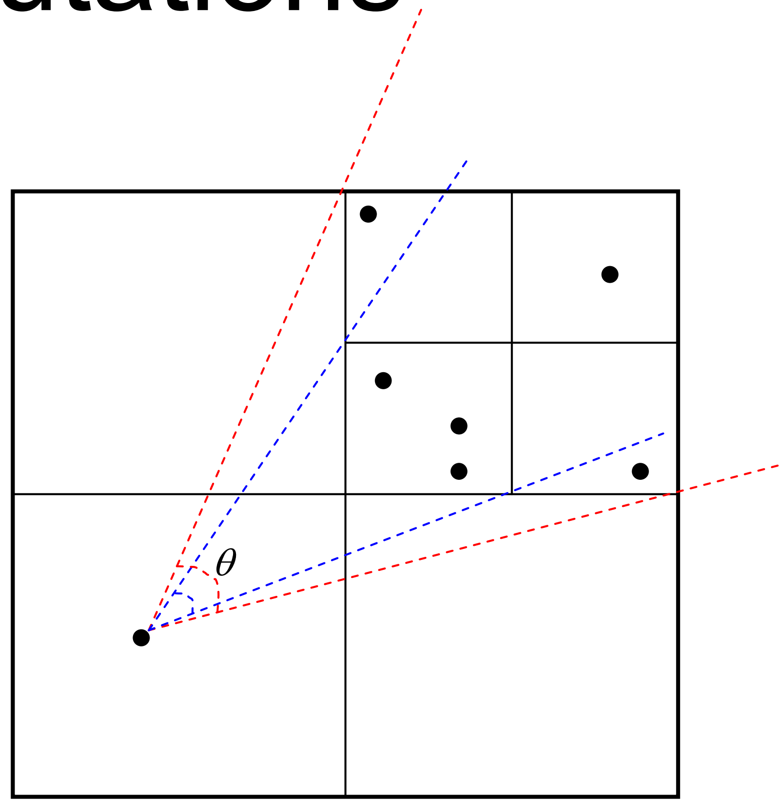
- A warm-up:
  Figure-8, 3-body problem

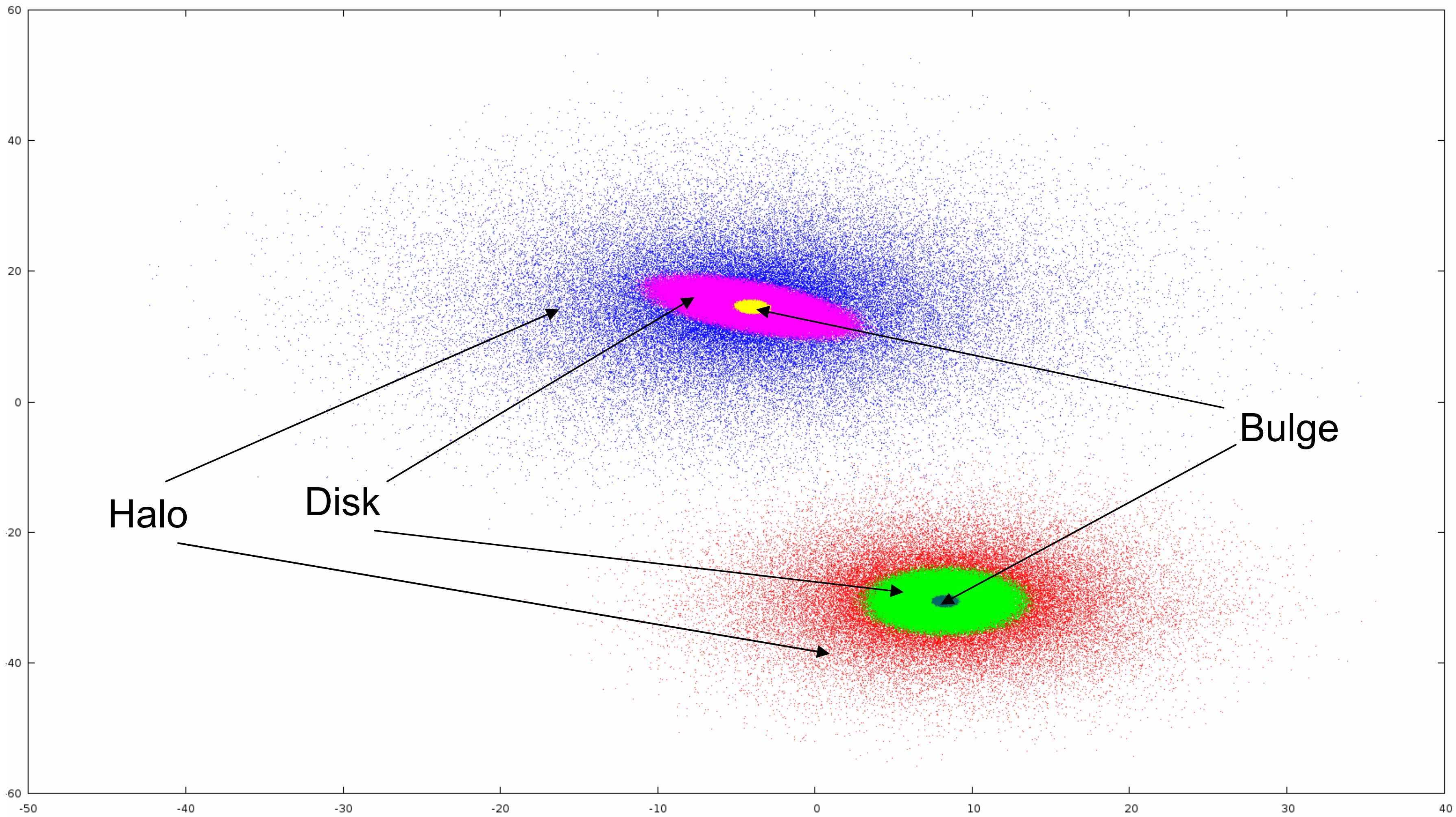


  - Use taylor 1.4.3 Integrator
  - Always truncate double precision to single precision at each step
  - With initial condition in 3D, but $z = v_z = 0, h = 0.05$
  - Do FMS: find a 20,000-step shadow (~160 loops)
  - Shadow distance: ~10^{-5}

Force Computations

- Direct force computation: $O(N^2)$
- In simulation:
  - Approximation: $O(N \cdot log(N))$
  - θ: 0.9 ~ 1.2 (Error: ~ 1%)
- In shadowing:
  - No approximation
  - Accurate floating-point summation
    - bucket summation
    - iFastSum: guaranteed correctly rounded, 5 times slower
  - Advantages
    - "physically correct"
    - LSODE works better (continuous force)





N = 1,024,000
SOFT = 0.01
h = 0.05
Time = 0.00 (initial condition)

Results and Discussions

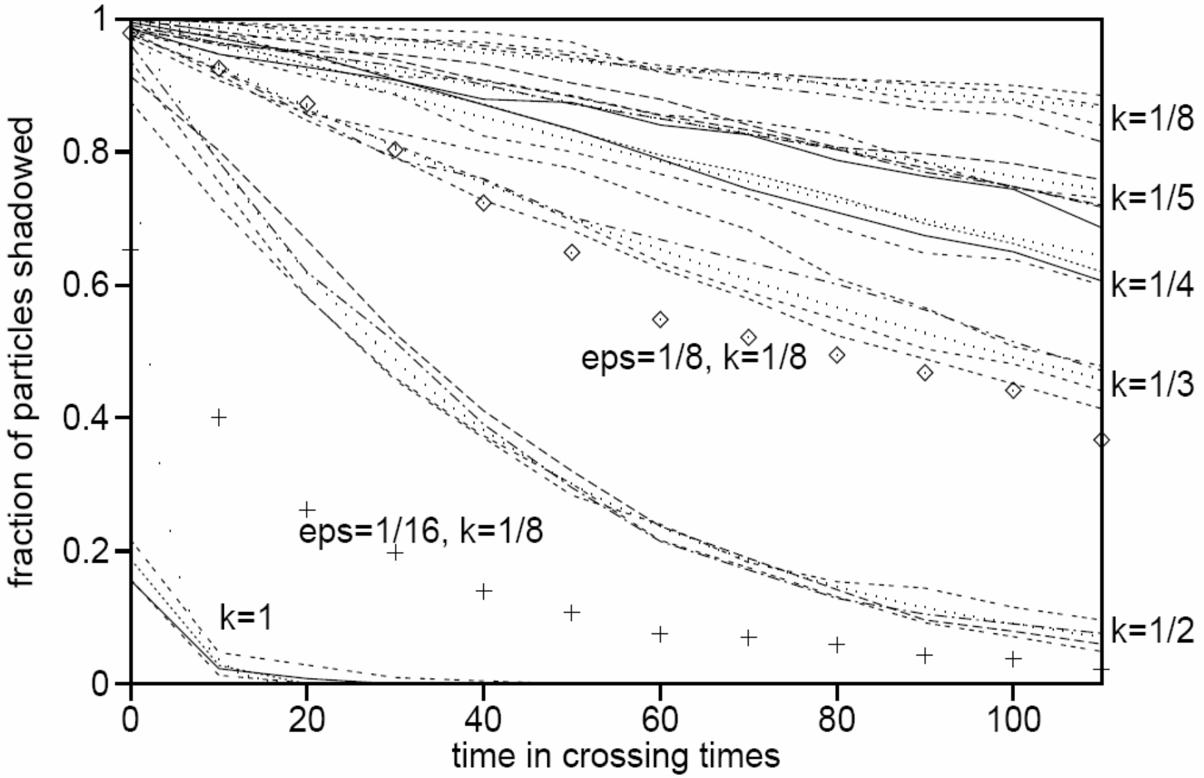
- Randomly pick particles for each mass
- Fixed-motion shadowing each of these particles
- Total Average Shadow Length: 9.82

Mass	Average Shadow Length
Disk 1	4.21
Disk 2	5.85
Bulge 1	3.06
Bulge 2	2.38
Halo 1	18.86
Halo 2	25.58

before the collision occurs

Results and Discussions

- Empirical results for FPS
  - Choose simulation parameters: $h^2 \propto \varepsilon^2 N^{1/3}$
  - For same k (when $\varepsilon \in [\sim 0.2, 1]$), we should get similar shadow length, where $h^2 = k^2 \varepsilon^2 (\frac{N}{100})^{1/3}$



Results and Discussions

- Try small systems
  - Scale N=10^6 simulation into N=10^3,10^4
  - Eliminate one galaxy
- Does FPS's formula work for FMS?
  - Do analysis for each type of particles
  - Switch to h to crossing time = R / V
    - 50% mass radius, R
    - Average speed, V
  - Count N for each type of particles
  - Compute k from $h^2 = k^2 \varepsilon^2 (\frac{N}{100})^{1/3}$
  - Estimate the average shadow length for the same k

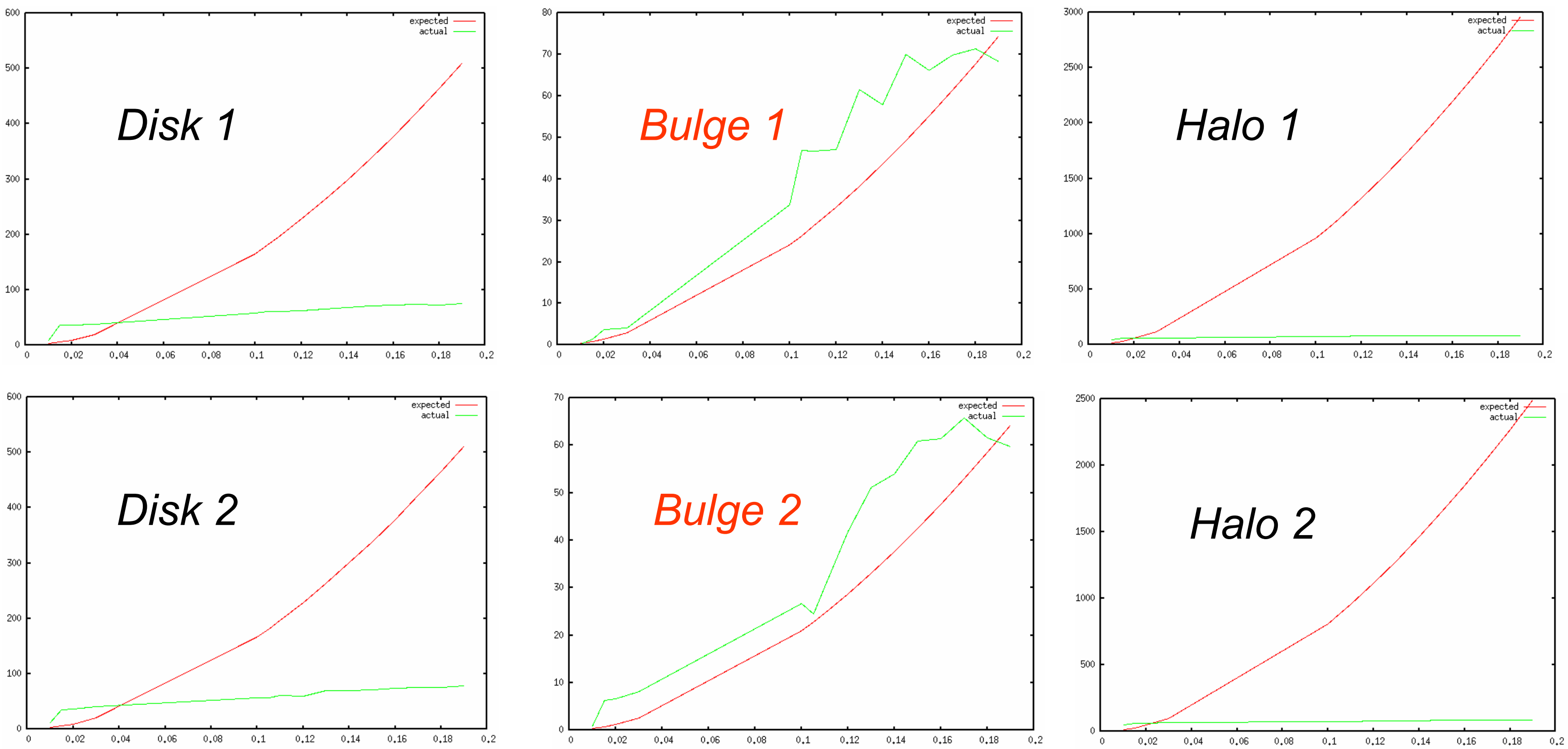
Results and Discussions

- Does FPS's formula work for FMS? (Cont'd)
  - $N=10^6$

Mass	Average	Expected
Disk 1	4.21	23.23
Disk 2	5.85	22.06
Bulge 1	3.06	3.23
Bulge 2	2.38	3.06
Halo 1	18.86	97.29
Halo 2	25.58	91.71

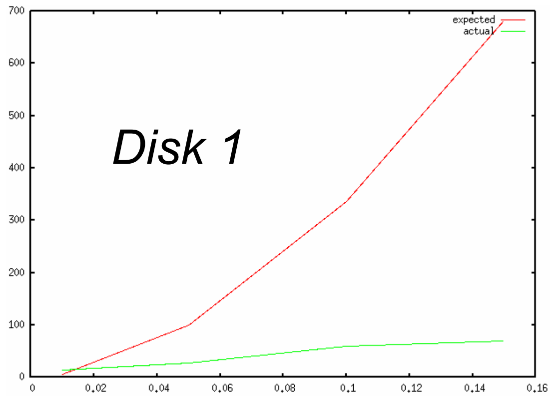
Results and Discussions

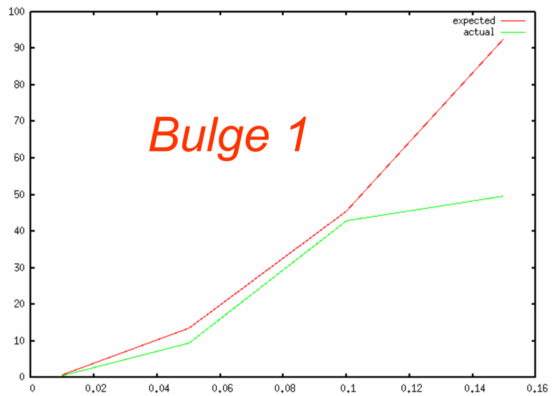
- Does FPS's formula work for FMS? (Cont'd)
  - N=10^3 (15 softenings each)

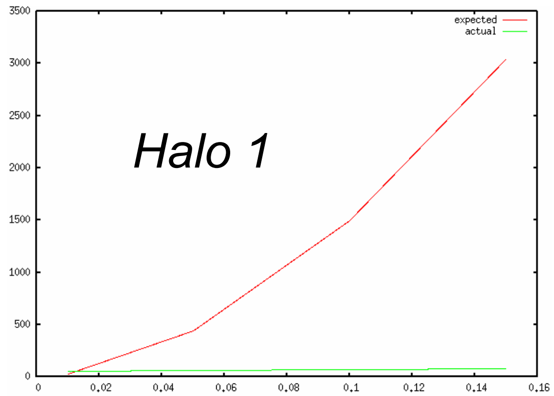


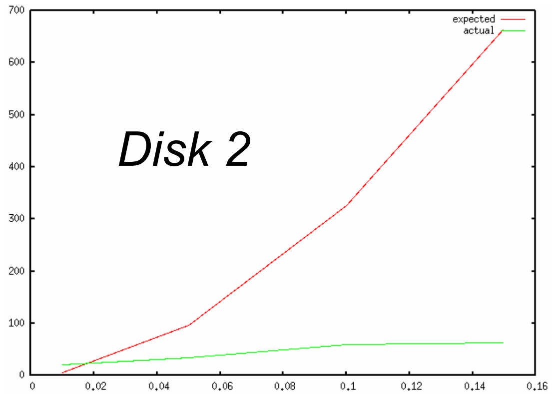
Results and Discussions

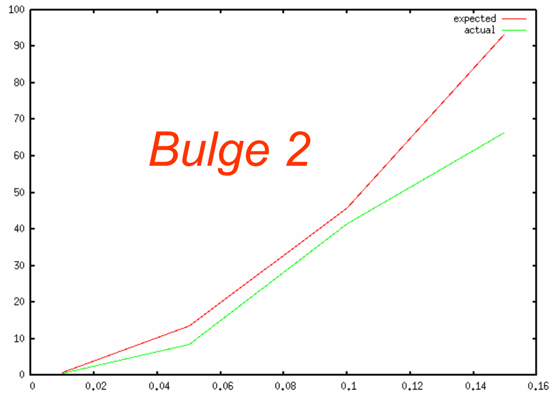
- Does FPS's formula work for FMS? (Cont'd)
  - N=10^4 (4 softenings each)

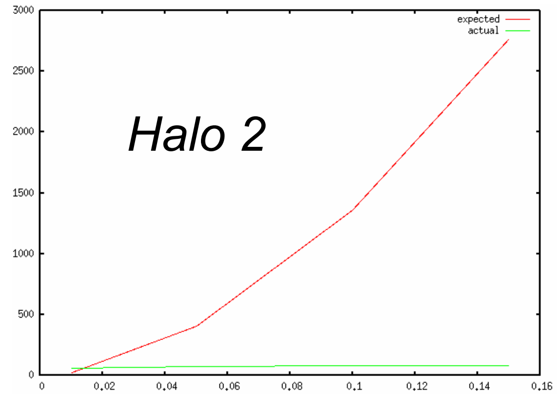












Results and Discussions

- Some conclusions
  - It works best for Bulge
  - For different N, the pictures look similar
  - Possible reasons
    - In FPS, all the particles in 1-unit sphere, like Bulge
    - Other particles slightly influence Bulge
    - Many Halo particles have full shadows (simulate longer? CPU?)
    - Disk is not distributed in a sphere
    - All Bulge-shadows end far before collision occurs, so got similar results when eliminating one galaxy
- Future work: how to make shadows longer?

Thanks!

Q & A