Model Predictive Control for Linear Systems with Interval and Stochastic Uncertainties*

Vladimir V. Dombrovskii
Tomsk State University, Tomsk, Russia
dombrovs@ef.tsu.ru

Elena V. Chausova
Tomsk State University, Tomsk, Russia
chau@ef.tsu.ru

Abstract

The work examines the problem of model predictive control for uncertain linear dynamic systems with intervally assigned parameters and multiplicative noise inputs. We use the model predictive control techniques based on linear matrix inequalities to get an optimal robust control strategy that provides the system with stability in the mean-square sense. The results are illustrated by an numerical example.

Keywords: linear dynamic system, interval uncertainty, stochastic uncertainty, model predictive control, convex optimization, linear matrix inequalities
AMS subject classifications: 93E20, 93C99

1 Introduction

The work examines the problem of model predictive control for uncertain linear dynamic systems containing both interval and stochastic uncertainties. The system uncertainties are expressed by the following assumptions. First, we assume that the system matrices depend on interval-valued parameters [13, 16, 17]. This is a quite realistic problem statement. In practice, many systems have uncertain parameters which are not known exactly, either because they are hard to measure or because the data necessary for a stochastic description are unavailable, etc. Frequently, we can only estimate intervals that bound the parameters as they vary in the course of time. In these cases, interval uncertainty description is the most suitable. Second, we allow that the system is disturbed by multiplicative noise inputs. These assumptions yield a model with a mix of interval and stochastic uncertainties. Such models can describe a large family of uncertain systems [22].

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Model predictive control (MPC) to be used in this paper is a popular control design method in system and control theory [1–4, 6–12, 14, 15, 18, 21]. The papers [3, 8, 14, 21] examine only the systems with polytopic uncertainty descriptions. The systems with stochastic uncertainty including multiplicative noises are studied in [2, 4, 7, 9–11, 15, 18]. MPC is based on the following concept [1, 6, 12]. At every time instant, we solve an optimization problem to calculate optimal future control inputs. Although more than one control move is calculated, only the first one is implemented. At the next sampling time, the state of the system is measured, and the optimization is repeated.

We use the MPC techniques based on linear matrix inequalities (LMIs) as introduced in [14]. LMIs have a wide range of applications in system and control theory [5, 19]. Some examples of applications are stability theory, model and controller reduction, robust control, system identification, and predictive control. LMI-based optimization problems have low computational complexity and can be solved numerically on-line very efficiently [20].

The study [14] is devoted to robust MPC for uncertain systems including the systems with polytopic uncertainties. We consider similar systems, except that our system is disturbed by multiplicative noise inputs. We consider the problem of designing, at each time step, a state feedback control law which minimizes a worst-case performance of an infinite horizon objective function. Using the approach proposed in [14], we formulate the original minimax optimization problem as a convex optimization problem involving LMIs. Solving it on-line, we get the optimal robust control strategy providing the system with stability in the mean-square sense. We present a numerical example to illustrate the results developed in this paper, and we offer some concluding remarks.

Our notations are standard. $\mathsf{E}\{\cdot\}$ denotes the expectation of a random variable (matrix), $\mathsf{E}\{\cdot|\cdot\}$ is the conditional expectation. $P > 0\,(P \geq 0)$ means that P is a positive definite (semi-definite) matrix. $\mathrm{Tr}(\cdot)$ is the trace of a matrix, $\mathrm{ch}\,\{\cdot\}$ denotes the convex hull. $A^\top$ is transpose of a matrix, and A^{-1} is inverse matrix.

2 Problem Statement

We consider discrete-time uncertain dynamic systems of the form

$$x(k+1) = \left(A_0(p(k)) + \sum_{j=1}^{n} A_j(p(k))w_j(k)\right)x(k) + \left(B_0(p(k)) + \sum_{j=1}^{n} B_j(p(k))w_j(k)\right)u(k), \qquad k = 0, 1, 2, \ldots. \quad (1)$$

In (1), $x(k) \in \mathbb{R}^{n_x}$ is the state of the system at time k, and $x(0)$ is assumed to be defined; $u(k) \in \mathbb{R}^{n_u}$ is the control input at time k; $w_j(k)$, $j = 1, \ldots, n$, are independent white noises with zero mean and unit variance, $\mathsf{E}\{w_i(k)w_j(s)\} = \delta_{ij}\delta_{ks}$, δ_{ij} is Kronecker delta symbol; $A_j(p(k)) \in \mathbb{R}^{n_x \times n_x}$, $B_j(p(k)) \in \mathbb{R}^{n_x \times n_u}$, $j = 0, \ldots, n$, are the state-space matrices of the system, and $p(k) \in \mathbb{R}^{n_p}$ is an uncertain parameter vector.

Suppose that the parameter vector $p(k)$ is subject to interval uncertainty; that is, we only know that $p(k)$ takes its values within an interval box $\boldsymbol{p}$ and any additional information is absent:

$$p(k) \in \boldsymbol{p}, \qquad k = 0, 1, 2, \ldots, \quad (2)$$

where $\boldsymbol{p} \in \mathbb{IR}^{n_p}$, $\mathbb{IR}$ is the set of the real intervals $\boldsymbol{x} = [\underline{x}, \overline{x}]$, $\underline{x} \leq \overline{x}$, $\underline{x}, \overline{x} \in \mathbb{R}$ [13,17].

The state-space matrices are assumed to depend affinely on $p(k)$. Then condition (2) can be replaced by the inclusion:

$$\big(A_0(p(k)), \dots, A_n(p(k)), B_0(p(k)), \dots, B_n(p(k)),\big) \in \Omega, \qquad k = 0, 1, 2, \dots, \tag{3}$$

where the set

$$\Omega = \mathrm{ch}\,\big\{\big(A_{01} \dots A_{n1}\ B_{01} \dots B_{n1}\big), \dots, \big(A_{0L} \dots A_{nL}\ B_{0L} \dots B_{nL}\big)\big\}\,,\ L = 2^{n_p},$$

is a polytope, and the uncertain state matrices lie in it for all time-varying $p(k) \in \boldsymbol{p}$. Hence, we can refer the uncertain system (1) to the class of polytopic systems.

Allowing two types of uncertainty in the system (1), we consider the following minimax performance objective:

$$\min_{u(k+i|k),\, i=0,\dots,m-1} \quad \max_{\big(A_0(p(k+i)),\dots,A_n(p(k+i)),B_0(p(k+i)),\dots,B_n(p(k+i))\big)\in\Omega,\ i\geq 0} J(k), \tag{4}$$

where

$$J(k) = \mathsf{E}\left\{ \sum_{i=0}^{\infty} \Big(x(k+i|k)^\top Q x(k+i|k) + u(k+i|k)^\top R u(k+i|k) \Big) \;\Big|\; x(k) \right\}.$$

This is the case of an infinite horizon model predictive control. Here $Q \in \mathbb{R}^{n_x \times n_x}$, $Q = Q^\top > 0$, $R \in \mathbb{R}^{n_u \times n_u}$, $R = R^\top > 0$, are symmetric weighting matrices; $u(k{+}i \mid k)$ is the predictive control at time $k{+}i$ computed at time k, and $u(k|k)$ is the control move implemented at time k; $x(k{+}i|k)$ is the state of the system at time $k{+}i$ derived at time k by applying the sequence of predictive controls $u(k|k), u(k+1|k), \dots, u(k+i-1|k)$ on the system (1), and $x(k|k)$ is the state of the system measured at time k, the exact measurement of the state of the system is assumed to be available at each sampling time k, that is $x(k|k) = x(k)$; m is the number of control moves to be computed, and it is assumed that $u(k+i|k) = 0$ for all $i \geq m$.

We solve the above problem by using the LMI-based MPC techniques as introduced in [14]. We apply the linear state-feedback control law

$$u(k+i|k) = Fx(k+i|k), \qquad i \geq 0, \tag{5}$$

where $F \in \mathbb{R}^{n_u \times n_x}$ is the state-feedback matrix. Then we derive an upper bound on the worst-case performance of the objective function $J(k)$ over the set Ω. At each time instant k, we calculate the state-feedback matrix of the control law (5) to minimize this upper bound. As is standard in MPC, only the first control move $u(k) = u(k \mid k) = Fx(k \mid k)$ is implemented, and we get the feedback control law for the current state $x(k)$. Then the state $x(k+1)$ is measured, and the optimization is repeated at the next sampling time $k+1$. As a result, we get the optimal robust feedback control strategy providing the system with stability in the mean-square sense,

$$\lim_{i\to\infty} \mathsf{E}\left\{x(k+i|k)x(k+i|k)^\top \mid x(k)\right\} = 0, \qquad k = 0, 1, 2, \dots, \tag{6}$$

for every trajectory of the system (1) in the polytope Ω.

3 Main Results

The following theorem gives the state-feedback matrix F in the control law (5).

Theorem 3.1 *Let $x(k) = x(k|k)$ be the state of the uncertain system* (1) *measured at sampling time k. Then the state-feedback matrix of the control law* (5) *which minimizes the upper bound on the worst-case value of $J(k)$ at time k is*

$$F = Y\, S^{-1}, \tag{7}$$

where the matrices $S = S^\top > 0$ and Y are obtained from the solution (if it exists) to the optimization problem

$$\min_{S=S^\top>0,\, Y,\, \gamma>0} \gamma \tag{8}$$

subject to

$$\begin{pmatrix} 1 & x(k|k)^\top \\ x(k|k) & S \end{pmatrix} \geq 0, \tag{9}$$

and

$$\begin{pmatrix} S & SA_{0l}^\top + Y^\top B_{0l}^\top & \dots & S\,A_{nl}^\top + Y^\top B_{nl}^\top & SQ^{1/2} & Y^\top R^{1/2} \\ A_{0l}S + B_{0l}Y & S & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A_{nl}S + B_{nl}Y & 0 & \dots & S & 0 & 0 \\ Q^{1/2}S & 0 & \dots & 0 & \gamma\, I & 0 \\ R^{1/2}Y & 0 & \dots & 0 & 0 & \gamma\, I \end{pmatrix} \geq 0, \tag{10}$$

$l = 1, \dots, L$, where I is a unit matrix, 0 is a zero matrix of suitable dimensions.

Proof: We assume that the predicted states of the system (1) satisfy

$$x(k+i+1|k) = \Big(A_0(p(k+i)) + \sum_{j=1}^{n} A_j(p(k+i))w_j(k+i)\Big)x(k+i|k)$$
$$+ \Big(B_0(p(k+i)) + \sum_{j=1}^{n} B_j(p(k+i))w_j(k+i)\Big)u(k+i|k),$$
$$\big(A_0(p(k+i))\dots A_n(p(k+i))B_0(p(k+i))\dots B_n(p(k+i))\big) \in \Omega, \quad i \geq 0.$$

By setting the control law (5), we arrive at the recurrent relation

$$x(k+i+1|k) = \Big(L_0(p(k+i)) + \sum_{j=1}^{n} L_j(p(k+i))w_j(k+i)\Big)x(k+i|k),$$
$$\big(A_0(p(k+i))\dots A_n(p(k+i))B_0(p(k+i))\dots B_n(p(k+i))\big) \in \Omega, \quad i \geq 0, \tag{11}$$

where $L_j(p(k+i)) = A_j(p(k+i)) + B_j(p(k+i))F$, $j = 0, \dots, n$. The objective function becomes

$$J(k) = \mathsf{E}\left\{ \sum_{i=0}^{\infty} \Big(x(k+i|k)^\top (Q + F^\top R\, F)x(k+i|k)\Big) \,\Big|\, x(k) \right\}. \tag{12}$$

Consider a positive definite quadratic function

$$V(k+i|k) = \mathsf{E}\left\{x(k+i|k)^\top P x(k+i|k)|x(k)\right\} = \mathrm{Tr}\left(PX(k+i|k)\right),$$

where $P \in \mathbb{R}^{n_x \times n_x}$, $P = P^\top > 0$; $X(k+i|k) = \mathsf{E}\{x(k+i|k)x(k+i|k)^\top|x(k)\} \geq 0$. Note that $V(k+i|k) \geq 0$ with $V(k+i|k) = 0$ if and only if $X(k+i|k) = 0$. At sampling time k, we suppose that V meets the condition

$$\begin{aligned} V(k+i+1|k) - V(k+i|k) &\leq -\mathsf{E}\left\{x(k+i|k)^\top (Q + F^\top R F) x(k+i|k)|x(k)\right\} \\ &= -\mathrm{Tr}\left((Q + F^\top R F) X(k+i|k)\right), \quad i \geq 0, \end{aligned} \tag{13}$$

for every trajectory of the predicted states (11). We get $\Delta V(k+i|k) = V(k+i+1|k) - V(k+i|k) \leq 0$ with $\Delta V(k+i|k) = 0$ if and only if $X(k+i|k) = 0$. Therefore, $V(k+i|k)$ is a strictly decreasing function and tends to zero as $i \to \infty$. Hence, $X(k+i|k) \to 0$ as $i \to \infty$. Thus, condition (13) guarantees the mean square stability (6) of the system at time k.

Summing (13) from $i = 0$ to $i = t$, we obtain

$$\begin{aligned} &V(k+t+1|k) - V(k|k) \leq \\ &\qquad \leq -\mathsf{E}\left\{ \sum_{i=0}^{t} \left(x(k+i|k)^\top (Q + F^\top R F) x(k+i|k) \right) \middle| \, x(k) \right\}. \end{aligned}$$

As $t \to \infty$, we have

$$-V(k|k) \leq -J(k).$$

Then

$$\max_{\left(A_0(k+i)\ldots A_n(k+i) B_0(k+i)\ldots B_n(k+i)\right) \in \Omega,\, i \geq 0} J(k) \;\leq\; V(k|k),$$

where $V(k|k) = \mathsf{E}\{x(k|k)^\top P x(k|k)|x(k)\} = x(k|k)^\top P x(k|k)$. Hence, V is a quadratic Lyapunov function. At the same time, this gives an upper bound on the worst-case of the objective function $J(k)$ in Ω. Thus, the goal (4) can be redefined to derive, at each time step k, a constant state-feedback control law (5) that minimizes the upper bound $V(k|k)$ on the worst-case $J(k)$.

We can derive

$$\begin{aligned} V(k+i+1|k) &= \mathsf{E}\{x(k+i+1|k)^\top P x(k+i+1|k)|x(k)\} \\ &= \mathrm{Tr}\left(\sum_{j=0}^{n} L_j(p(k+i))^\top P \, L_j(p(k+i)) X(k+i|k) \right). \end{aligned}$$

Then, condition (13) may be written as

$$\mathrm{Tr}\left(\left(\sum_{j=0}^{n} L_j(p(k+i))^\top P \, L_j(p(k+i)) - P + Q + F^\top R F \right) X(k+i|k)) \right) \leq 0,$$

$$P > 0,$$

and we get

$$\sum_{j=0}^{n} L_j(p(k+i))^\top P \, L_j(p(k+i)) - P + Q + F^\top R F \leq 0, \qquad P > 0,$$

where $L_j(p(k+i)) = A_j(p(k+i)) + B_j(p(k+i))F$, $j = 0, \ldots, n$. Defining $P = \gamma S^{-1}$ and $F = YS^{-1}$, where $S = S^\top > 0$, $\gamma > 0$, we obtain

$$\sum_{j=0}^{n} \Big(A_j(p(k+i)) + B_j(p(k+i))YS^{-1}\Big)^\top \gamma S^{-1} \Big(A_j(p(k+i)) + B_j(p(k+i))YS^{-1}\Big) - \gamma S^{-1} + Q + (YS^{-1})^\top RYS^{-1} \leq 0.$$

Pre- and post-multiplying by S and dividing by γ yield

$$\sum_{j=0}^{n} \Big(A_j(p(k+i))S + B_j(p(k+i))Y\Big)^\top S^{-1} \Big(A_j(p(k+i))S + B_j(p(k+i))Y\Big) - S + \gamma^{-1}SQS + \gamma^{-1}Y^\top RY \leq 0$$

or

$$S - \begin{pmatrix} C_0(p(k+i)) \\ C_1(p(k+i)) \\ \vdots \\ C_n(p(k+i)) \\ Q^{1/2}S \\ R^{1/2}Y \end{pmatrix}^\top \begin{pmatrix} S & 0 & \ldots & 0 & 0 & 0 \\ 0 & S & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & S & 0 & 0 \\ 0 & 0 & \ldots & 0 & \gamma I & 0 \\ 0 & 0 & \ldots & 0 & 0 & \gamma I \end{pmatrix}^{-1} \begin{pmatrix} C_0(p(k+i)) \\ C_1(p(k+i)) \\ \vdots \\ C_n(p(k+i)) \\ Q^{1/2}S \\ R^{1/2}Y \end{pmatrix} \geq 0, \tag{14}$$

where $C_j(p(k+i)) = A_j(p(k+i))S + B_j(p(k+i))Y$, $j = 0, \ldots, n$.

Now we use the following result of the LMI theory which converts some non-linear inequalities to LMI form. This is referred to as the non-strict Schur complement [5, 19].

Let $C(x) = C(x)^\top > 0$, $A(x) = A(x)^\top$ and $B(x)$ depend affinely on x. Then the LMI

$$\begin{pmatrix} A(x) & B(x) \\ B(x)^\top & C(x) \end{pmatrix} \geq 0$$

is equivalent to the matrix inequalities

$$A(x) \geq 0, \quad A(x) - B(x)C(x)^{-1}B(x)^\top \geq 0.$$

Apply the Schur complement to (14). If $S = S^\top > 0$, $\gamma > 0$, then (14) is equivalent to

$$\begin{pmatrix} S & C_0^\top(p(k+i)) & \ldots & C_n^\top(p(k+i)) & SQ^{1/2} & Y^\top R^{1/2} \\ C_0(p(k+i)) & S & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ C_n(p(k+i)) & 0 & \ldots & S & 0 & 0 \\ Q^{1/2}S & 0 & \ldots & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & \ldots & 0 & 0 & \gamma I \end{pmatrix} \geq 0, \tag{15}$$

which is affine with respect to $\big(A_0(p(k+i))\dots A_n(p(k+i))B_0(p(k+i))\dots B_n(p(k+i))\big)$. Hence, condition (15) becomes

$$\begin{pmatrix} S & SA_{0l}^\top + Y^\top B_{0l}^\top & \dots & S\,A_{nl}^\top + Y^\top B_{nl}^\top & SQ^{1/2} & Y^\top R^{1/2} \\ A_{0l}S + B_{0l}Y & S & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A_{nl}S + B_{nl}Y & 0 & \dots & S & 0 & 0 \\ Q^{1/2}S & 0 & \dots & 0 & \gamma\, I & 0 \\ R^{1/2}Y & 0 & \dots & 0 & 0 & \gamma\, I \end{pmatrix} \geq 0,$$

$$l = 1, \dots, L. \quad (16)$$

This is the LMIs in S, Y, and γ. Hence, any $S = S^\top > 0$, Y, and $\gamma > 0$ satisfying the above LMIs gives an upper bound $V(k|k)$ and guaranties the mean-square stability of the system.

Finally, let us introduce an additional constraint

$$V(k|k) = x(k|k)^\top Px(k|k) \leq \gamma.$$

We substitute $P = \gamma S^{-1}$, where $\gamma > 0$, $S = S^\top > 0$, and divide by γ, which yields

$$1 - x(k|k)^\top S^{-1} x(k|k) \geq 0.$$

Using the Schur complement, the above inequality can be rewritten as

$$\begin{pmatrix} 1 & x(k|k)^\top \\ x(k|k) & S \end{pmatrix} \geq 0. \quad (17)$$

Hence, the minimization problem for computing the upper bound on $V(k|k)$ is equivalent to

$$\min_{S=S^\top>0,\, Y,\, \gamma>0} \gamma$$

subject to LMI constraints (16) and (17). At the same time, the feedback matrix is given by $F = YS^{-1}$, which is required to be proved. ■

4 Numerical Example

We consider the system described by the following model:

$$x(k+1) = \Big(A_0(\alpha(k)) + A_1(\beta(k))w(k)\Big)x(k) + \Big(B_0(\alpha(k)) + B_1(\beta(k))w(k)\Big)u(k),$$
$$k = 0, 1, 2, \dots,$$

where the state matrices are given by

$$A_0(k) = \begin{pmatrix} 1 & 0.1 \\ 0 & 1 - \alpha(k) \end{pmatrix}, \qquad A_1(k) = \begin{pmatrix} \beta(k) & 0 \\ 0 & 0.9 \end{pmatrix},$$

$$B_0(k) = \begin{pmatrix} 0.5\alpha(k) & 0 \\ 0 & 0.3 \end{pmatrix}, \qquad B_1(k) = \begin{pmatrix} \beta(k) & 0 \\ 0 & 0 \end{pmatrix},$$

$$\alpha(k) \in [0.1, 0.7], \qquad \beta(k) \in [0.2, 0.8],$$

$w(k)$ is a white noise with zero mean and unit variance. We have a system of the form (1) with the parameter vector $p(k) = (\alpha(k);\ \beta(k))^\top$. The weighting matrices of the performance objective are

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad R = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}.$$

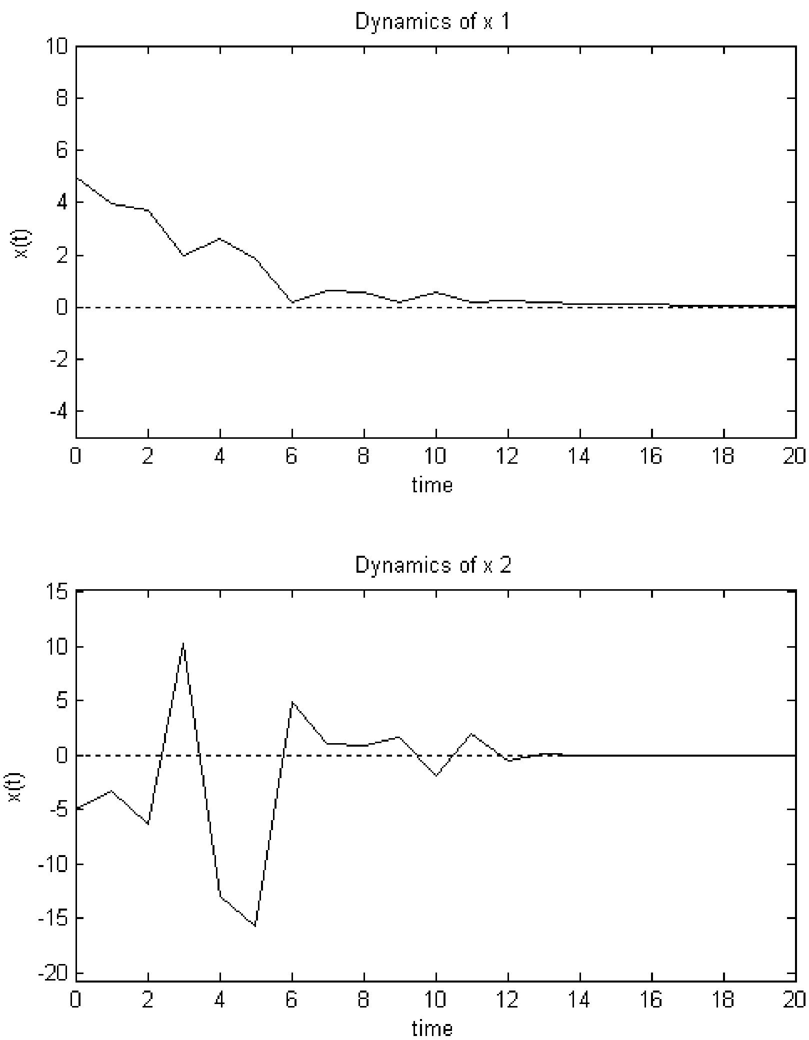

Figure 1: System dynamics under optimal control strategy with $x(0) = (5,\ -5)^\top$

For simulation, we used the LMI toolbox from MATLAB that provides various routines for solving LMI's [19]. Fig. 1 illustrates results from our simulation with the initial state $x(0) = (5,\ -5)^\top$. We can see that the system approaches the desired zero trajectory.

5 Conclusions

We have examined the problem of model predictive control for uncertain linear dynamic systems with interval-valued parameters and multiplicative noise inputs. Using the LMI-based MPC approach, we have reduced the original minimax optimization problem to a convex optimization problem involving LMIs that can be solved efficiently. As a result, we determined the optimal robust control strategy providing the system with stability in the mean-square sense. We have illustrated the developed results by a numerical example.

References

[1] A. Bemporad. Model-based predictive control design: New trends and tools. *Proc. 45th IEEE Conference on Decision and Control.* San Diego, CA, 2006, pp. 6678–6683.

[2] A. Bemporad and S. Di Cairano. Model-predictive control of discrete hybrid stochastic automata. *IEEE Trans. Automatic Control,* 2011, Vol. 56, No. 6, pp. 1307-1321.

[3] A. Bemporad and M. Morari. Robust model predictive control: A survey. *Robustness in Identification and Control.* Springer: Berlin, 1999, pp. 207–226.

[4] D. Bernardini and A. Bemporad. Scenario-based model predictive control of stochastic constrained linear systems. In *Proc. Joint 48th IEEE Conf. on Decision and Control and 28th Chinese Control Conf.,* Shanghai, P.R. China, 2009, pp. 6333–6338.

[5] S. Boyd, L.EL Ghaoui, E. Feron, and Balakrishnan. *Linear Matrix Inequalities in System and Control Theory.* SIAM, vol. 15 of Studies in Applied Mathematics. Philadelphia, PA, 1994.

[6] E.F. Camacho and C. Bordons. *Model Predictive Control.* Springer-Verlag, London, 2004.

[7] M. Cannon, B. Kouvaritakis, and X. Wu. Model predictive control for systems with stochastic multiplicative uncertainty and probabilistic constraints. *Automatica,* 2009, Vol. 45, No. 1, pp. 167–172.

[8] A.F. Cuzzola, J.C. Geromel, and M. Morari. An improved approach for constrained robust model predictive control. *Automatica,* 2002, Vol. 38, No. 7, pp. 1183–1189.

[9] V.V. Dombrovskii, D.V. Dombrovskii, and E.A. Lashenko. Predictive control of random-parameter systems with multiplicative noise. Application to investment portfolio optimization. *Automation and Remote Control,* 2005, Vol. 66, No. 4, pp. 583–595.

[10] V.V. Dombrovskii and T.U. Obyedko. Predictive control of systems with Markovian jumps under constraints and its application to the investment portfolio optimization. *Automation and Remote Control,* 2011, Vol. 72, No. 5, pp. 989–1003.

[11] E.L. Ghaoui. State-feedback control of systems with multiplicative noise via linear matrix inequalities. *Systems and Control Letters,* 1995, No. 24, pp. 223–228.

[12] K.S. Holkar and L.M. Waghmare. An overview of model predictive control. *International Journal of Control and Automation International Journal of Control and Automation,* 2010, Vol. 3 No. 4, pp. 47-63.

[13] L. Jaulin, M. Kieffer, O. Didrit, and E. Walte. *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control and Robotics.* Springer-Verlag, London, 2001.

[14] M.V. Kothare, V. Balakrishnan, and M. Morari. Robust constrained model predictive control using linear matrix inequalities. *Automatica,* 1996, Vol. 32, No. 10, pp. 1361–1379.

[15] B. Kouvaritatakis, M. Cannon, and V. Tsachouridis. Recent developments in stochastic MPC and sustainable development. *Annual Reviews in Control,* 2004, No. 28, pp. 23-35.

[16] V. Kreinovich. Interval uncertainty as the basis for a general description of uncertainty: A position paper. Downloadable from `http://www.cs.utep.edu/vladik/2012/tr12-38.pdf`.

[17] R.E. Moore, R.B. Kearfott, and M.J. Cloud. *Introduction to Interval Analysis.* SIAM, Philadelphia, PA, 2009.

[18] J.A. Primbs and C.H. Sung. Stochastic receding horizon control of constrained linear systems with state and control multiplicative noise. *IEEE Trans. Automatic Control,* 2009, Vol. 54, No. 2, pp. 221–230.

[19] C. Scherer and S. Weiland. Linear Matrix Inequalities in Control, 2005, `http://www.dcsc.tudelft.nl/~cscherer/lmi/notes05.pdf`.

[20] L. Vandenberghe and V. Balakrishnan. Algorithms and software for LMI problems in control. *IEEE Control Syst. Mag.,* 1997, pp. 89–95.

[21] Z. Wan and M.V. Kothare. Efficient robust constrained model predictive control with a time varying terminal constraint set. *Systems and Control Letters,* 2003, Vol. 48, pp. 375–383.

[22] F. Wang and V.Balakrishnan. Robust steady-state filtering for systems with deterministic and stochastic uncertainties. *IEEE Trans. Signal Processing,* 2003, Vol. 51, No. 10, pp. 2550–2558.