Detecting and Locating Curved Cracks in Thin Plates by Lamb Wave Reflection: Validated Geometric Approach

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The practical problem. In many practical situations, it is extremely important to be able to detect hidden cracks (and other possible faults) in aerospace and other structures. One way of testing the structural integrity of such structures is by using ultrasonic waves. For thin plates, Lamb waves (which direct the energy along the plate) are especially useful. To use these waves, we set up a transmitter T and (one or several) receivers R, and try to extract, from the signals measured by the receivers, the information about the possible cracks and other faults.

This extraction is not easy.

Lamb waves are difficult to control. In many practical problems – e.g., in radar detection – we detect objects by sending waves and processing the signals measured by receivers. In most such situations, waves travel by air, so we can easily control them: we can use an antenna to reflect the generated waves and thus, to focus the waves in the desired direction; similarly, we can use an antenna to gather waves coming from different directions into a single point and thus, amplify the received signal.

In contrast, Lamb waves travel *inside* the plate. There is no easy way to reflect them without placing reflectors inside the plate – i.e., without introducing additional faults.

Reconstructing the fault location and shape from the Lamb wave measurements is a computationally difficult problem. Propagation of Lamb waves is described by known PDEs. In general, there exist efficient computational techniques for solving PDEs. However, these techniques cannot be easily applied to the problem of fault location: Indeed, these techniques usually assume that we know the boundary conditions, but in the fault location problem, the location of the boundary is exactly the problem.

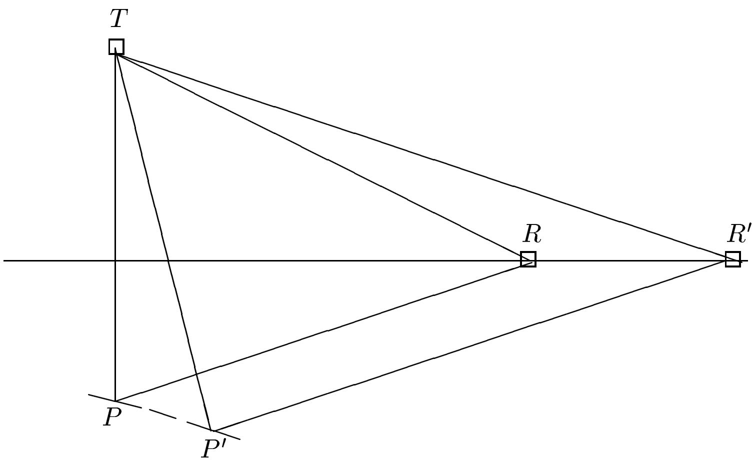
As a result, reconstructing the location and the shape of a crack from the measurement results is a very difficult task. Even for simple faults like edge cracks near the rivet holes, the corresponding methods have been developed only recently [1]. The existing methods enable us, at best, to predict, for a given size and orientation of the crack, what the signal will be. Since the corresponding formulas are very complex, the only way to detect the location and size from the measurement results is to compare these results with theoretical predictions corresponding to different crack sizes and locations. This is not practical. We need a method which will transform the known signal into the information about the location and size of the crack, and the complexity of the existing formulas prevents us from doing it.

Geometric approach. One way to avoid this complexity problem is to use a *geometric* approach, when instead of trying to fully capture the complex physics of Lamb waves, we only use general geometry-related properties of wave propagation.

What was known before: detection of linear cracks. In [2], the geometric approach has been successfully used to detect linear cracks. Detection of such cracks is based on the following idea. In an ideal (faultless) plate, a Lamb wave goes directly from the transmitter to the receiver. The signal emitted by the transmitter at time t reaches the receiver at time $t + t_0$, where $t_0 = d_0/v$, $d_0 = \overline{TR}$ is the distance between the transmitter T and the receiver R, and v is the wave speed. The signal detected by the receiver has the same shape as the signal sent by the transmitter, but shifted in time by t_0. In particular, if the transmitter emitted a short pulse train, the signal registered by the receiver will consist of the same short pulse train – occurring at a later moment of time.

What happens when the plate contains a crack? A crack reflects the wave. As a result, if a transmitter sends a pulse train at a certain moment of time t, this pulse train first reaches the receiver directly, at the moment $t + t_0$, and then another copy of this pulse train reaches the receiver indirectly, after first hitting the crack and then being reflected by the crack. So, if the plate contains a crack, the receiver will receive the signal consisting of *two* pulse trains: the earlier pulse train at time $t + t_0$ which comes directly from the transmitter, and a later pulse train coming at a time $t + t_1 = t + d_1/v$, where $d_1 = \overline{TP} + \overline{PR}$ is the total path of the reflected signal (P is the point on the crack where the signal was reflected). By measuring the time t_1 between the emitted pulse train and the second detected pulse train, we can thus determine the total path D of the reflected signal as $d_1 = v \cdot t_1$.

This information can be used to locate the crack. Indeed, for the (unknown) reflection point P, we know the sum $\overline{TP} + \overline{PR}$ of the distances from two known points: T and R. It is a known geometrical fact that for any given two points T and R, the set of all points P with a given sum $\overline{TP} + \overline{PR}$ is an ellipse. Due to Snell's law describing wave reflection, the angle between the incoming wave and the crack must be the same as between the crack and the outcoming wave. Due to the properties of an ellipse, we can conclude that the crack is tangent to this ellipse at the reflection point P. The paper [2] shows how we can use this fact to compute the coordinates of the straight line crack.



Our main results. In practice, cracks are not exactly straight, they are curved. There is no known way to describe a shape of a realistic (curved) crack by a finite-parametric formula :-(We therefore need techniques for detecting and locating generic cracks. Such geometric techniques are proposed in this paper.

We also provide guaranteed bounds for the crack location. The inaccuracy of the fault location is caused by the inaccuracy of measuring the arrival time. The main component of the time measurement error is the discretization error. For this error, we know its upper bound Δt. Thus, when the measured time value is $\widetilde{t} = k \cdot \Delta t$, the only information about the actual time t is that t must belong to the interval $[\widetilde{t} - \Delta t, \widetilde{t} + \Delta t]$.

The exact values of t would lead to the exact location of the point in a fault. Since the input data has interval uncertainty, we cannot get the exact location; instead, we use interval computations to describe the area that is guaranteed to contain a point from the fault.

Preliminary results show that this method indeed works well on actual data.

Acknowledgments. This work was supported by AFOSR grant F49620-00-1-0365. We are thankful to Ajit Mal and to the anonymous referees for useful discussions.

References.

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