Fast or Tight Propagation of Univariate Taylor Coefficients

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It is by now well understood that, for functions defined by computer programs Taylor, polynomials in several variables can be recovered from families of univariate polynomials. We observe here that by adapting an idea of Kronecker it is possible to get by with a single univariate polynomial of rather high order.

Hence the evaluation of higher derivatives can be based on the propagation of univariate Taylor polynomials though the sequence of arithmetic functions and elementary intrinsics defining the function at hand. This can be done with very regular memory access patterns, and using Newton's method or other fixed point iterations everything can be reduced to a small number of convolutions.

Thus the convolution becomes the work horse of higher order differentiation. It may be performed either optimal in the interval sense with a quadratic complexity or fast in essentially linear time using FFT. Either way one achieves significant improvements on the basic recurrences due to Moore, but so far we have not found a "best of both worlds" i.e., fast and tight propagation method.