Interval Methods in Digital Signal Processing

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Interval methods represent a relatively new research direction in digital signal processing. Though, in the closely related field of controls there has been much work that can also be applied to signal processing. In general interval methods provide a way for providing verification of computations or as an optimization procedure. Interval arithmetic provides a method for determining how numerical errors scale as a result of implementing algorithms on computational machines of various word lengths and number representation. The tracking of numerical errors can be exploited in signal processing through the use interval arithmetic since the computations are done on various types of computers. Computing systems used range from dedicated processors using fixed point arithmetic with short word lengths to supercomputers using floating point arithmetic with very large word lengths.

The estimation of system parameters from noisy data represents another important topic in signal processing. If the system has feedback or is nonlinear then the associated objective function to be optimized can be nonconvex. This would require global optimization methods to insure that convergence to the absolute optimal is achieved. Interval methods, a deterministic optimization method, advantage over other global optimization methods is its ability to find the global optimum of nonconvex differentiable or non-differentiable objective functions. It represents the method to attempt first if the one has no knowledge of where the global optimum might exist on the parameter space.

This talk will focus on the research that was done by the author in applying interval methods to digital signal processing. Work which covers both optimization and analysis. In optimization, we will discuss the use of interval methods for solving the sinusoidal parameter estimation problem. Research on adaptive systems using interval analysis to validate the results and to monitor stability and errors will also be discussed. The talk will end with a discussion on problems and prospects of using interval methods in signal processing.

The sinusoid parameter estimation problem consist of determining the maximum-likelihood estimates of sinusoid parameters from a signal that consists of sinusoids and additive noise. We will present three algorithms that integrate interval methods for global optimization with procedures that decompose the problem into smaller ones. The interval method used is a global optimization technique that is based upon the branch and bound principle. More specifically, decomposition of the problem is accomplished via the expectation-maximization algorithm and the grouped coordinate descent algorithm. Although a formal proof of convergence is not addressed, the performance of the algorithms from simulations was shown to be superior to the popular iterative quadratic maximum likelihood (IQML) method.

An adaptive system is one that can adapt to a changing environment through optimization of the systems parameters. Thereby, its objective function can vary as a function of time. Adaptive filtering algorithms can have problems converging to the optimal parameters due to numerical errors and tracking a time-varying objective function. These errors are manifested through instability of the algorithm, arithmetic precision caused by finite word length of the processor, slow convergence and the ability to track a time varying minima. The use of interval arithmetic yields a better performing algorithm by:

- Monitoring certain parameters of the optimization algorithm to eliminate instabilities caused by mathematical operations of numbers of very different order of magnitude.
- Bounding the results.
- Bounding the parameter space to insure the algorithm converges to a stable filter.
- In conjunction with heuristics or evolutionary strategies for fast convergence to the global minimum.

Many of the adaptive filtering applications will be implemented on a finite word length machine that are optimized to implement multiply-sum operations fast and efficiently, i.e. digital signal processors. These machines implement arithmetic operations, basic to signal processing algorithms, of addition, subtraction, and multiplication. This implies that a reduced version of interval arithmetic operations can be implemented and is feasible for development on field programmable gate arrays (FPGA's). This could further lead to developing hardware for an adaptive system that adapts to both the parameters and the structure (word length, filter order, and the number of inputs and outputs). To change or adapt the systems structure, it must have the ability to monitor performance and for the hardware to evolve based on changing conditions of its operating environment.

The problems associated with applying interval methods to signal processing is the additional time required for arithmetic operations and the lack of awareness amongst the signal processing community. To overcome the problem of speed is to develop dedicated hardware that is optimal for interval arithmetic operations, e.g. FPGA's.

References

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