The Application Fields of the RejectIndex Parameter in Interval Methods for Global Optimization

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Suppose that for a given function $f : X \subseteq \mathbb{R}^n \to \mathbb{R}$, the global minimum f^*, or an approximation of it $(\hat{f})$ is a priori known. It may be obtained for example using a local search algorithm or during the execution of the B&B algorithm for global minimization. Let us define a parameter $pf^*(Y)$ called RejectIndex as:

$$pf^*(Y) = \frac{f^* - \underline{F}(Y)}{\overline{F}(Y) - \underline{F}(Y)} \in [0, 1], \; Y \subseteq X.$$

Capitals denote intervals, underline and overline the lower and upper bounds, respectively. The inequality $f^* \geq \underline{F}(Y)$ holds, since pf^* is evaluated for intervals Y for which $F(Y)$ contains the global minimum value.

This parameter was designed mainly based on the following: traditionally an interval Y with the minimal value of $\underline{F}(Y)$ was considered as the best candidate to contain a global minimum. However, usually the larger the interval Y, the larger the over-estimation of the range $f(Y)$ obtained in $F(Y)$. Therefore a box could be considered as a good candidate to contain a global minimum just because it is larger than others. In order to compare subintervals with different size fairly we normalize the distance between f^* and $\underline{F}(Y)$.

The idea behind pf^* is simply that we expect the over-estimation to be nearly symmetric, i.e. the over-estimation $\overline{F}(Y) - \overline{f}(Y)$ above $f(Y)$ is closely equal to the over-estimation $\underline{f}(Y) - \underline{F}(Y)$ below $f(Y)$, for small sub-intervals containing a global minimizer point (that is at the same time a stationary point). Hence, for such intervals Y the relative place of the global optimum value inside the $F(Y)$ interval should be high, while for intervals far from global minimizer points pf^* must be small. Obviously, there are exceptions, and there exist no theoretical proof that pf^* would be a reliable indicator of nearby global minimizer points.

Based on these ideas, several application fields of pf^* and its variants were investigated. The talk plans to summarize them in a systematic way. The

RejectIndex has been previously used in parallel interval B&B algorithms as a predictor of the computational work associated to boxes in the work tree. In [1] it has been shown that using RejectIndex an almost perfect work load balance for parallel implementations of the Interval B&B algorithms can be obtained. It can also be applied to improve the multisection decision rule to achieve better overall efficiency (see [2]), since in the neighbourhood of minimizer points the intervals must be subdivided into more subintervals to decrease the number of function evaluations.

For hard to solve global minimization problems it may be an option to drop the guaranteed reliability (at least short term), and to get rid of those generated subintervals that can hardly contain global minimizer points. This can be done again on the basis of the pf^* values. According to our experiences, this technique could be an effective measure, and problems unsolved by traditional interval methods could be solved by interval minimization methods using the related heuristic rejection rules [3, 4]. To keep the reliability of interval methods, the rejected subintervals can be written into an output file for a possible later processing.

The fourth way for utilizing the RejectIndex is to use it in the decision which one from the list of candidate subintervals is to be chosen for the next subdivision. The interval selection rule is a very sensitive part of the B&B method, since over two decades no new paradigm was suggested in this field. Casado and coworkers suggested a new (not necessarily reliable) interval selection method based on pf^*, and reported improvements in efficiency in [5]. It was shown in [6] that with known global minimum value or with a good approximation of it a different new interval selection rule ensures the convergence of the minimization procedure to global minimizer points, and that it improves again the efficiency of the algorithm substantially.

Two further papers address additional issues on RejectIndex: the article of Kreinovich and Csendes shows ([7]) that the interval selection rule based on pf^* is optimal in certain sense; and Markót and coworkers suggest new algorithm parameters similar to RejectIndex for constrained minimization problems [8].

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