Ramon E. Moore (1929–2015)

Ramon Edgar (Ray) Moore was born on December 27, 1929 in Sacramento, California. The oldest of six children, Ray looked after his siblings when his mother Rose went to work. His extended family included many aunts, uncles, and cousins. Ray learned marksmanship from an uncle, later organizing a high school rifle team and leading it to a western regional championship. An early interest in chemistry (reportedly including the manufacture of nitroglycerine in an upstairs bedroom) was sparked by a chemistry set from Ray's father Edgar. Life for the Moore family was strongly influenced by the Great Depression; Edgar and Ray fished the Sacramento river to catch dinner during a time when money was tight for everyone in the community. But Ray read voraciously and his intellectual acuity did not go unnoticed by his teachers. Driven to leave Sacramento for a bigger world, he earned a scholarship to the University of California at Berkeley at age 16.

Upon receiving the A.B. degree in physics from Berkeley in 1950, Ray accepted a position in the Terminal Ballistics Lab at the US Army's Aberdeen Proving Ground in Maryland. After some months around exploding bazooka shells, Ray took an opportunity to transfer to Aberdeen's computing center. There he joined a group of two dozen mathematicians who were exploring the use of early electronic computing machines such as ENIAC, EDVAC, ORDVAC, and the Bell relay computer. Ray was present when the ENIAC team ran their first program, designed to calculate π to 1000 digits. He learned programming concepts such as flowcharts and subroutines from John von Neumann, and mathematical analysis from others in the group.

After three years at Aberdeen, Ray transfered to the University of California Radiation Laboratory at Livermore, acquiring mathematical and computer programming skills related to problems in hydrodynamics and nuclear physics. A highlight of this experience was a two-week trip to the IBM headquarters in New York to test a major computer program on the world's most powerful machine: the IBM 701. Ray's program, written in octal, required the computer's entire capability: all of the drums, tapes, card readers and punchers, and random access memory that it had. As Ray stood at the operator's console, a refrigerator-sized power supply unit went up in smoke when the program began to execute. Ray responded by jumping up and down and shouting "*It works! It works!!*" While IBM sent for new hardware, Ray and his assistants had a few days to relax and enjoy the sights of the big city. The program later ran successfully.

In 1956, Ray accepted a position at Lockheed's Palo Alto Research Laboratories. There he met Lockheed consultant and Stanford University professor George Forsythe. After seeing a paper in which Ray had developed all the tools needed for an interval method of solving ordinary differential equations, Forsythe persuaded Ray to enter the doctoral program in mathematics at Stanford and served as dissertation advisor. The paper became the basis for Ray's doctoral dissertation; he received his Ph.D. in mathematics from Stanford in 1963.

Subsequently Ray held a series of academic positions at the University of Wisconsin at Madison (1965–1981), the University of Texas at Arlington (1982–1986), and the Ohio State University (1986–2000). He also held visiting professorships at Stockholm, Sweden (1967), Oxford, England (1974), Karlsruhe, Germany (1975), and Freiburg, Germany (1980). He consulted for The World Bank (1978–1979), Lockheed MSC (1985–1986), Sun Microsystems (2000–2002), and Michigan State University (2004). Ray was twice recipient of the Alexander von Humboldt Foundation U.S. Senior Scientist Award (1975 and 1980).

The first of Ray's books was his seminal work *Interval Analysis*, published by Prentice-Hall in 1966. His other books included *Computation and Theory in Ordinary Differential Equations* (1970), *Mathematical Elements of Scientific Computing* (1975), *Methods and Applications of Interval Analysis* (1979), *Computational Functional Analysis* (1985 and 2007), the edited volume *Reliability in Computing* (1988), and *Introduction to Interval Analysis* (2009). Ray also published over 30 journal papers and technical reports. His retrospective article *The Dawning* (Volume 5, Issue 4 of *Reliable Computing*) describes in his own words a striking moment of mathematical discovery.

Along with his wife Adena, Ray even drafted a book called *Number Pairs* aimed at introducing interval concepts to youthful audiences. Ray fervently shared his stories, interests, and experiences with his family. One summer he combined attendance at a mathematics conference with a memorable two-week trip to Ireland with Adena and their two children, Devin and Claire. Ray's broad curiosity led him to investigate such fields such as psychology and neuroscience, and he deeply enjoyed classical music and the outdoors. Relishing words as well as numbers, he engaged in a rewarding weekly correspondence with poet and lifelong friend Daniel J. Langton.

Ray Moore is remembered as a powerfully creative intellect, a founder of interval mathematics, a brilliant educator, a supportive mentor, and a warm friend. He passed away on April 1, 2015. The profound manner in which Ray touched the lives of others is evinced by the following sequence of personal recollections.

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Ray conceived of interval analysis in 1958, the year he joined Lockheed Research Labs Palo Alto California. From the start his vision was for much more than just the definition and implementation of computing with intervals, or interval arithmetic. Nevertheless, very quickly, he developed the first practical general purpose interval program (to solve ordinary differential equations) and his programmers produced its computer implementation. Ray's demonstrated success developing interval algorithms and computing with them established the utility of both interval analysis and general purpose interval algorithms as well as an existence proof that computing some interval algorithms can be practical.

Entire careers of various mathematicians and computer scientists were significantly stimulated and influenced, if not owed to Ray's early pioneering work, including successful computations, lectures, and publications. Ray never oversold computing with intervals. For example, he was quick to reveal that interval Gaussian Elimination gave surprisingly poor (too wide) bounds on errors when solving systems of linear equations. Ray was always candid about this and other interval arithmetic shortcomings. However, together with Ray's encouragement, the challenges posed by these difficulties, when contrasted with spectacular interval computing successes, only added to the lure that prompted researchers that followed Ray to become steeped in further developing interval analysis as well as interval algorithms and their implementation.

In spite of the fact that Ray always appeared relaxed and somewhat laid back, his achievements belie his demeanor. His copious productivity was both impressive and revolutionary.

Finally, Ray was a great friend and colleague to us and all who knew and worked with him. We miss him.

Eldon Hansen and Bill Walster

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The influence made by Ramon Moore's activity on the development of interval analysis worldwide can hardly be overestimated, and his two main deeds contributed to the new science especially strongly. These were Moore's invention of the very term "interval analysis" and his publication, in 1966, of the first systematic book *Interval Analysis* devoted to the new mathematical discipline. I should say that the origin and further development of the wide mathematical researches on interval analysis in Russia and, more broadly, in the Soviet Union, is closely connected with Moore's book. Although sporadic inventions of what we now call "interval methods" were made by several people in Russia since the 1920s (V.M. Bradis, L. Kantorovich, etc.), they had not led to the start of scholarly mathematical research in interval analysis. In particular, they had not resulted in self-sufficient scientific schools, since the 'interval inventors' were either lone persons or had few progeny in this field.

The most powerful Russian group of interval researchers (to which I myself belong) emerged in Novosibirsk near the end of the 1960s under the auspices of Nikolai Yanenko, a prominent mathematician who headed the Institute of Theoretical and Applied Mechanics in Novosibirsk Academgorodok at that time. Nikolai Yanenko is widely known as "the father of the fractional steps method," a popular decomposition method for the numerical solution of partial differential equations. But he was a broadly thinking intellectual who got interested in the adjacent fields of the computational and applied mathematics in general. In 1968, during one of his foreign trips, Nikolai Yanenko got acquainted with Moore's book *Interval Analysis* and then bought it. He was deeply impressed by the book's clear and fresh language, new and interesting problem statements, and instructive applications of interval techniques. Yanenko decided to commence research on interval analysis in his own institute, and organized a "special interval group" headed by Yurii Shokin. That was the origin of our Siberian Scientific School on interval analysis and its applications. At present, after four decades of intensive development, it includes Yuri Shokin, Sergey Kalmykov, Ziyaviddin Yuldashev, Boris Dobronets, Irene Sharaya,† and a great many others. And the main impetus in its creation was made by Ramon Moore's intellectual activity and his pioneering book.

Sergey P. Shary
Novosibirsk, Russia

†*Editors' note*: We are sorry to report the loss of another member of the interval community, Irene Sharaya (1962–2015). Irene's bio can be found in Vol. 20 of *Reliable Computing.*

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Ray first touched me as a fresh Assistant Professor at the University of Nebraska-Lincoln in 1975. I was beginning to teach numerical analysis and to do research in Taylor series methods for the numerical solution of differential equations. I was led to Ray's *Interval Analysis* book by his programmed generation of Taylor coefficients now known as automatic differentiation. Ray's recurrence relations paralleled those of Y.F. Chang, my Taylor series mentor, but Ray's explanations were much more understandable.

At the time, I was struggling to learn enough about IBM 360 floating point arithmetic to teach it in an assembly language course and to understand results my numerical analysis students were experiencing, so Ray's Interval Analysis struck me as eminently logical. I was drawn in by his exposition; why would one want to compute any other way?

After moving to Marquette University in Milwaukee in 1978, I made many trips to Madison, then to Arlington and to Columbus, to visit and listen to Ray. He struck me as relaxed, with a humble demeanor and respect for all. He always was more interested in the work of others than in describing his own work. He exemplified an attitude I later summarize as, "It is hard to learn with an open mouth."

In working with Ray, Bill Walster, Baker Kearfott, and several others on a project in the late 90's to uncover and show-case interval "killer applications," I converted Shiresh, an economics masters student, to intervals. When he gave his first public seminar extolling the merits of intervals for Value At Risk analysis, someone asked the usual "But aren't they slow?" question. Shiresh stopped, just stared at the questioner in utter disbelief, "Why would you **want** to get the **wrong answer?**" When I told the story to Ray, he burst out laughing, "Ah, the faith of a new convert."

Ray's gentle mentorship and friendship inspired me to a concern for correctness. I spent one entire summer in the basement of the Computer Science building in Madison finding and removing over 100 programming errors of the form $0 \in [1, 2] \times 10^{-16}$ (in IBM double precision). In floating point arithmetic, I would have thought I had the right answer; in interval arithmetic, it was guaranteed to be **wrong**, and I could hunt down and correct the causes. Today, I would try harder not to introduce the errors in the first place, but techniques I learned from Ray helped me remove them and deliver a robust software package.

The ideas Ray promoted as his life's work form the basis of the recently-adopted IEEE 1788 Standard for Interval Arithmetic.

Like many good ideas, interval arithmetic claims several early pioneers. Like many good ideas, in interval arithmetic, one of the early pioneers casts an especially long shadow by his strong influence on three generations of followers and their academic children. That's Ray Moore. We miss him.

George Corliss
Milwaukee, Wisconsin

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When I started my PhD thesis in 1991, my supervisor Eric Walter asked me to read Ray's book *Interval Analysis* (Prentice-Hall, 1966). Eric wanted me to study what interval analysis could do to solve nonlinear bounded-error estimation problems. It was a very good idea for two reasons. First, interval analysis was the right tool for nonlinear estimation; second, this book positively influenced my entire career as a researcher. Even now I often open the book and am impressed by the number of important ideas that were proposed, such as automatic differentiation, guaranteed integration, the interval Newton method, methods for solving nonlinear problems, etc.

Early during my PhD thesis, through my supervisor, I had some contacts with Ray about computing the inner and outer approximations of feasible parameter sets. As we can read in his paper "Parameter sets for bounded-error data" (Mathematics and Computers in Simulation, 1992), he early found an elegant way to bracket the solution set between two subpavings. This idea gave birth to what later became "set inversion."

I was very proud of this collaboration and, with Eric Walter, decided to invite Ray to my PhD defense as a reviewer. I will never forget all the interesting discussions we had about the applications of interval analysis to estimation and control when he came to Paris for those few days.

From the beginning of my PhD and probably until the end of my research activities, my main objective is to show that interval analysis, as proposed by Ray, is one of the most important theories for the solution of modern applications where reliability is needed. I think I have been able to convince some of my colleagues and PhD students, maybe thanks to some nice applications we had in the domain of robotics and control theory.

Luc Jaulin
Brest, France

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In honor of Ramon Moore, I would like to briefly recount the history of interval arithmetic at Karlsruhe.

In 1966 my university got a new computer: an Elektrologika X8. I was the director of the computer centre. Also in 1966 Ramon Moore's book on interval arithmetic was published. It contained many fascinating and promising ideas. Because the X8 did not provide for directed rounding, we studied its hardware and, jointly with the manufacturer, added the downward rounding operation. Upward rounding was done using $\text{roundup}(x) = -\text{rounddown}(-x)$. This hardware extension could be done in three hours. Within a year our collaborator Hans Wilm Wippermann implemented an ALGOL extension providing a new data type and operations for intervals. Jürgen Herzberger implemented the elementary functions for the types real and interval. Goetz Alefeld, Nicolaos Apostolatos, myself and others developed and studied early applications.

In December 1967, Hans Stetter organized a meeting on interval arithmetic in Vienna. Ramon Moore and several of us from Karlsruhe were invited. We all talked about early achievements with interval arithmetic. For most of us it was the first personal meeting with Ramon Moore. At that time he was a professor at the Computer Science department at the University of Wisconsin.

A few months later I got an invitation from Barkley Rosser and Louis B. Rall to visit the Mathematics Research Centre at the University of Wisconsin in Madison. I accepted and stayed for the year 1969–1970. I used my time to work on a mathematical foundation of computer arithmetic, not just for real numbers and intervals, but also for intervals over the complex numbers, vectors and matrices over the real and complex numbers. The results were published in several papers and finally summarized in a book, *Grundlagen des Numerischen Rechnens — Mathematische Begründung der Rechnerarithmetik*, published in 1976 [12].[1]

[1] Citations refer to the list at the end of the paper "High Speed Associative Accumulation of Floating-point Numbers and Floating-point Intervals."

Simultaneously my collaborators Goetz Alefeld and Juergen Herzberger wrote the book *Einführung in die Intervallrechnung*. An English translation *Introduction to Interval Computations* was published by Academic Press in 1983.

When IBM Germany in nearby Boeblingen became aware of the activity at Karlsruhe, I was invited to visit their Research Centre at Yorktown Heights in 1972–73. There Willard Miranker organized and led a seminar on computer arithmetic. William Kahan was invited for about the same time. So we were both frequent speakers in that seminar.

In 1976 I got a call for a Professorship in Computer Science at the University at Paderborn, located in the hometown of the successful German computer company Nixdorf Computers. Early microprocessors appeared on the market at this time. So I visited Heinz Nixdorf and suggested joint development of a small microprocessor based computer. He agreed and funded the project jointly with the Deutsche Forschungsgemeinschaft. This project forced me to stay at Karlsruhe, and I needed all the well educated collaborators of my institute. Within three years we developed an extension of PASCAL called PASCAL-XSC together with a number of routines for standard problems of numerical analysis. PASCAL-XSC provides data types and operations for all the mathematical spaces considered in my book. Reinhard Kirchner and Michael Neaga implemented the compiler, and Juergen Wolff von Gudenberg the arithmetic operations and the elementary functions for the types real and interval. Problem solving routines with automatic result verification for a major number of standard problems of numerical analysis were developed by Edgar Kaucher and Siegfried Rump. Reinhard Kirchner and I exhibited our computer at the Nixdorf booth of the Hannover Fair in April 1980.

Heinz Nixdorf donated a number of these computers to the University at Karlsruhe. This allowed us to decentralize a part of the programming education beginning with the summer semester 1980. At that time batch processing with punch cards and punch tapes was standard everywhere else. The PC first appeared on the market in 1982.

As a reward, Heinz Nixdorf became an Honorary Senator of the Universitaet Karslruhe. Our contact with his company ended abruptly with his sudden death.

I visited the IBM Research Centre again in 1978–79. Willard Miranker and I worked on the manuscript for the book *Computer Arithmetic in Theory and Practice* (Academic Press, 1981) [13]. This visit to Yorktown Heights was the birth of a fruitful and long-lasting close collaboration and friendship with Willard.

In May 1980 Prof. Karl Ganzhorn, at that time the Director of IBM Europe, asked for a demonstration of our PASCAL-XSC system. Siegfried Rump provided an impressive one. For all examples our computer delivered tight bounds for the solution instead of murky approximations. After half an hour Prof. Ganzhorn said, "This will become part of every IBM computer." Thus, still in the same year, a ten-year collaboration between IBM and my institute was initiated. All the work for IBM had to be done under strict confidentiality. Our project manager at IBM was Hartmut Bleher. In 1983, Siegfried Rump changed within the project from his university position at Karlsruhe to the IBM

Laboratory at Boeblingen.

In close analogy to our PASCAL-XSC we developed a corresponding FORTRAN extension for their S/370 architecture called ACRITH-XSC together with a large library of routines for all kinds of standard problems in numerical analysis. For all applications the computer delivered close bounds on the solution. Wolfgang Walter, Michael Metzger, and Lutz Schmidt were leading figures for the implementation of the language. Walter Kraemer and Klaus Braune solved another major task: the implementation of the elementary functions for real and complex number and interval arguments. A first release, ACRITH, was published in 1983. On this occasion I got an enthusiastic letter from Ramon Moore. Several other releases followed. The language extension ACRITH-XSC followed in 1990. All collaborators in the project worked with very high motivation. The frequent presence of Willard Miranker at Karlsruhe was an essential contribution to this. The commercial success of ACRITH-XSC was modest. In 1982 the PC entered the market and, around 1984, several workstations. I was asked by IBM at Austin, Texas, to work out a proposal and estimate the cost for transferring our software to the IBM RS 6000. The letter was returned. IBM had become aware that too much personnel was on board. So a generous payoff program was launched in which all our contact persons in IBM left the company. Our once close cooperation with IBM ended abruptly.

In 1982 IBM sponsored a seminar at the IBM Research Centre at Yorktown Heights at which all speakers besides Louis Rall and Willard Miranker came from my institute. A proceedings volume *A New Approach to Scientific Computation*, edited by Miranker and myself, was published by Academic Press in 1982 [14]. In 1982 Willard Miranker received the prestigious Humboldt U.S. Senior Scientist Award, which Ramon Moore had already received in 1980.

In 1986 I got a call to become the director of the Institute for Informatik at the Hahn–Meitner Institute at Berlin in combination with a professorship in Applied Mathematics at the Freie Universitaet Berlin. The governor of the state of Baden-Württemberg did not want to let me go to Berlin. So I got a generous financial offer. This supported more work on the applications of interval arithmetic with all the experienced collaborators of the IBM projects and a number of newly selected former students. A book published by Academic Press in 1993 gives an impression of the work done at that time. A selection of contributions is listed at the end of my recollection here.

Since April 1999 I have been emeritus at Universitaet Karlsruhe. This does not mean that I stopped working. In September 2000 another major conference around interval arithmetic, SCAN 2000, was held at Karlsruhe. One proceedings volume, *Scientific Computing, Validated Numerics, Interval Methods* (Kluwer) was edited by Walter Kraemer and Juergen Wolff von Gudenberg; another one, *Perspectives on Enclosure Methods* (Springer), by Rudolf Lohner, Axel Facius, and myself.

In 1999–2000 and in 2002–03 I spent several months at the Electrotechnical Laboratory at Tsukuba in Japan. I used the time to prepare the book *Advanced Arithmetic for the Digital Computer: Design of Arithmetic Units* (Springer, 2002) [17]. Then I wrote *Computer Arithmetic and Validity: Theory, Imple-*

mentation and Applications (De Gruyter, 2007; second edition 2013) [19]. After that I spent much time working in an international committee for developing a standard for interval arithmetic. I published several papers related to this activity.

For many years interval arithmetic was understood as a calculus for closed and bounded real intervals. In my early books [12, 13] I extended it to the usual product spaces of computation like complex numbers, real and complex interval vectors, and interval matrices. To achieve this with high accuracy an exact dot product was needed and provided. With my more recent book [19] I was able to extend interval arithmetic from bounded to unbounded intervals. The resulting calculus is free of exceptions. It remains so if the bounds are restricted to a floating-point screen.

Early in 2014, John Gustafson sent me the manuscript for his book *The End of Error* [4] and asked for comment. Reading the book became a big surprise. It gives interval arithmetic a final push by extending it from closed intervals to just connected sets of real numbers. These now can be closed, open, half-open, bounded or unbounded. This opens new areas of application and a path to better results. As the book was written in an informal style, I felt challenged to develop a more formal axiomatic approach to the material [21]. John Gustafson welcomed this enthusiastically.

Ramon Moore always appeared as a quiet, calm, and unassuming person. His influence on the development of reliable computing in all parts of the world, however, is extraordinary and overwhelming. What he once initiated with his early book on interval arithmetic in 1966 and stimulated for more than 50 years finally led to a complete, closed, and exception-free calculus with a very high potential for the future of scientific computing. It puts him in a unique position in the world of scientific computing. Intervals bring the continuum on the computer which allows results with guarantees. This is a fundamental difference from conventional floating-point arithmetic where this property is not available.

Ulrich Kulisch
Karlsruhe, Germany

Postscript by Prof. Kulisch. Use of the XSC-languages greatly simplified programming and led to more easily readable code. This enabled relatively rapid development of routines for many applications where the computer delivers close bounds for the solution — results out of reach for conventional floating-point arithmetic. I mention here some papers in the book [1]. The first contributions give an overview of the XSC-languages. They include Rolf Hammer, Michael Neaga, and Dietmar Ratz writing on PASCAL-XSC, Wolfgang Walter writing on ARITH-XSC, and Christian Lawo writing on C-XSC. Contributions on applications follow: "Automatic Differentiation and Applications" by Hans-Christoph Fischer, "Numerical Quadrature by Extrapolation with Automatic Result Verification" by Rainer Kelch, "Numerical Integration in two Dimensions with Result Verification" by Ulrike Storck, "Verified Solution of Integral Equations with Applications" by Hans-Juergen Dobner, "Enclosure Methods for Linear and Nonlinear Systems of Fredholm Integral Equations of the Second Kind" by Wolfram Klein, "A Step Size Control for Lohner's Enclosure Algorithm for Ordinary Differential Equations with Initial Conditions" by W. Rufeger and E. Adams, "Interval Arithmetic in Staggered Correction Format" by Rudolf Lohner, "Multiple Precision Computation with Result Verification" by Walter Kraemer, "Verification of Asymptotic Stability for Interval Matrices and Applications in Control Theory" by Beate Gross, and "Numerical Reliability of MHD Flow Calculations" by Wera U. Klein.

Other early applications are already discussed in the book [14]: "Solving Algebraic Problems with High Accuracy" by Siegfried M. Rump, "Evaluation of Arithmetic Expressions with Maximum Accuracy" by Harald Boehm, "Solving Function Space Problems with Guaranteed Close Bounds" by Edgar Kaucher, and "Features of a Hardware Implementation of an Optimal Arithmetic" by Ulrich Kulisch and Gerd Bohlender. Similarly, the book [16] included "Highly Accurate Verified Error Bounds for Krylov Type Linear System Solvers" by Axel Facius, and "Nonsmooth Global Optimization" by Dietmar Ratz.

Beyond the names already mentioned, a number of colleagues received doctorates at Karlsruhe for their work on interval arithmetic. They include Otto Mayer (1968), Bruno Lortz (1971), Christian Ullrich (1972), Rudi Klatte (1975), Dalcidio Claudio (1978), Kurt Gruener (1979), Thomas Teufel (1984), Guenter Schumacher (1989), Hubert Erb (1991), Andreas Knoefel (1991), Alexander Davidenkov (1992), Stefan Geoerg (1993), Manfred Schlett (1994), Matthias Hocks (1995), Peter Schramm (1995), Andreas Wiethoff (1996), Peter Januschke (1998), Chin Yun Chen (1998), Werner Hofschuster (2000), Stefan Wedner (2000), Stefan Dietrich (2002), and Norbert Bierlox (2003).

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I first met Ray in 2005 while urging him to publish a second edition of his 1985 book *Computational Functional Analysis*. My argument was that this elegant little text would be perfect for an audience of engineers two decades after its initial publication for mathematicians and computer scientists. He readily agreed and so did his original publisher Ellis Horwood. We worked on the new version of "*CFA*" for many months before I learned of Ray's legendary status as a founder of Interval Analysis. He simply never mentioned it until I asked. At that point I began to fathom Ray's dedication to interval mathematics and how much he esteemed the interval community. He used the Internet to quietly monitor developments in the field and was extremely pleased with what he saw. This ultimately led to our second book project, *Introduction to Interval Analysis*, with Baker Kearfott.

But I will surely remember Ray most fondly as a storyteller — the best I've ever met. Our weekly communications quickly grew past our writing projects and we traded stories about the professional world, the classroom, and life. He described his early days as a college student ("When I started, I was thinking chemistry, but soon got sick of the smells ..."), his natural affinity for mathematics ("Any kind of math was always easy ..."), and how he sometimes annoyed his Stanford professors with challenging questions while declining to take notes during lecture. Of course, he had some funny classroom encounters of his own; my favorite concerned a certain architecture student who announced during Ray's calculus lecture that she was *an artist* who would never need the subject for anything! There were fascinating anecdotes about early contacts with brilliant persons such as "Johnny" von Neumann, who could gain profound insights into systems of partial differential equations by rapidly skimming reams of tractor-feed computer paper, and deeply appreciative recollections of Lockheed's "cracker-jack programmers" who helped with the first implementations of outward rounding. One story ended with the words "My eyes were opened to the fact that computational mathematics was exciting, there was very very much to do."

Michael Cloud
Okemos, Michigan

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I knew Ray Moore for over 50 years and we were directly or indirectly in touch for essentially all of that time. This provides a lot of material for a memoir from which I will extract a small amount mainly of personal interest. I will give my first impression of him, his contributions to my career and those of many others, and our last communications.

First, a little background to our first meeting early in October, 1964. Before 1955 or so, serious numerical calculations were done with hand or electric powered desk calculators. By the middle of that decade, electronic computers capable of a dozen or so arithmetic operations per second were available. While individual steps of "hand calculations" were available for immediate inspection, they were generally unavailable or simply too numerous to check when performed by a digital computer. This made the accuracy of the final result of a program which ran for even a modest amount of time somewhat mysterious. This problem was considered serious (and bound to get worse) by a number of numerical analysts, including myself and J. Barkley Rosser, Director of the Mathematics Research Center at the University of Wisconsin. He asked me to organize conferences on the topic of error in digital computation in the Fall of 1964 and the Spring of 1965. This resulted in my first meeting with Moore.

In the course of assembling a panel of experts, one of my colleagues said that George Forsythe at Stanford had had a student who "worked on roundoff error." Since rounding is an insidious source of computational error, I invited said former student, Ramon Moore. From his name, I thought he might belong to the rich Spanish heritage of California. Instead, I met a tall, blue-eyed person of Irish extraction whose name was RAY-mon, not Ramón.

In those days, lectures at conferences were given using overhead projectors. Moore put his detailed transparencies on the projector and looked at them intently while reading from them in a rather soft voice. It was a lecture style that required the undivided attention of the audience. For one, I heard what he said. As is typical of an authentic genius, he got to the heart of the matter and made it seem obvious. I took away the concepts of interval analysis and algorithmic differentiation (often called automatic differentiation for historical reasons).

The talks at the conference dealt with methods for specific types of problems such as linear systems and differential equations, the ones which dealt with computer arithmetic itself were Moore's interval arithmetic and multiple precision arithmetic, the latter presented by Robert L. Ashenhurst. While multiple precision is useful in many situations, it has the potential to overwhelm the resources of the computer. (At the time of these conferences, most computers were limited to storage of a few thousand numbers of fixed length.) By contrast, interval arithmetic uses the set of numbers already available on the computer but requires controlled rounding of results of operations and functions. This required special implementation at the time but is now standard. The main feature of interval arithmetic was to insure the exact result of the computation lies between two ordinary machine numbers of given precision, the challenge is for this interval to be as small as possible. As already noted, this has motivated a large number of researchers, myself included, and has led to a huge body of publications.

It should be mentioned that interval arithmetic subsequently led to some disappointments. Because of its lack of additive and multiplicative inverses, interval arithmetic is unsuited to many splendid algorithms. In spite of this, some thought they could just "plug in" intervals for numbers in their computations and blamed the lack of satisfactory results on interval arithmetic instead of themselves.

The second thing that impressed me in Moore's lecture was his use of recursive generation of Taylor coefficients to obtain interval bounds for truncation errors. Everyone my age had been taught in school that one obtained formulas for derivatives from formulas for the functions being differentiated. Evaluation of the derivative then involved first obtaining the formula for the derivative, then evaluating it. From Moore's presentation, it dawned on me that derivatives could be evaluated from the evaluation of the function itself; furthermore, it was not necessary to have a formula for the function, just an algorithm (e.g., a computer program) based on differentiable operations and functions. Thus, the idea for algorithmic differentiation was there. This was especially important to me in connection with my interest in the derivative-intensive Newton's method for nonlinear systems of equations, where values of first derivatives were required for the Jacobian matrix, and second derivatives for the Kantorovich existence theorem and error estimates. The concept of being able to obtain rigorous error bounds which included discretization, truncation and roundoff error was very satisfying to me from a mathematical standpoint. Algorithmic differentiation took a while to catch on, but has played a substantial role in the careers of others as well as mine.

Ray came to Wisconsin shortly after the 1964–65 conferences. His subsequent activities are described in the other memoirs in this issue. I will turn from my first to my last contacts with him. He had informed me some time previously that he was seriously ill, but in August, 2014 he told me he was feeling well and looking forward to seeing his recently born grandson Wyatt. Toward the end of 2014, I sent him some ideas I had on interval arithmetic, my first work on the subject in many years. On January 18, 2015, he sent me very useful comments and suggestions but also the ominous statement that all treatments had ceased and he had about two months to live. He said he accepted the situation and was happy with his 85 years. He was dedicated to family, friends and mathematics and was a great inspiration to me as well as others. The interval community and those who were close to Ray miss him.

Louis Rall
Madison, Wisconsin