Applications of Fuzzy Measures and Intervals in Finance*

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Abstract

Computational intelligence techniques are very useful tools for solving problems that involve understanding, modeling, and analyzing large data sets. One of the numerous fields where computational intelligence has found an extremely important role is finance. More precisely, the problem of selecting an investment portfolio to guarantee a given return, at a minimal risk, have been solved using intelligent techniques such as support-vector machines, neural networks, rule-based expert systems, and genetic algorithms. Even though these methods provide good and usually fast approximation of the best investment strategy, they suffer some common drawbacks including the neglect of the dependence among criteria characterizing investment assets (i.e. return, risk, etc.), the ignorance of the interdependence among assets, and the assumption that all available data are precise and certain. To face these weaknesses, we suggest the use of utility-based multi-criteria decision making setting and fuzzy integration over intervals.

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1 Introduction

Given the pervasive nature of computer science, virtually all areas have had to deal with enormous amounts of data. These data alone do not provide much information if they cannot be analyzed, understood, and used to extend knowledge. The strength of computational intelligence is to give a wide variety of techniques that can be used to process, model and understand these datasets.

One of the fields where computational intelligence has been extremely useful is finance. Over the past four thousand years, finance has been studied from various perspectives [8] ranging from basic arithmetic to probabilistic techniques and stochastic modeling, and machine learning approaches. Numerous problems in the area of finance use computational techniques (e.g. data mining, machine learning, stochastic differential equations), to solve problems such as option pricing and portfolio management. The former deals with how to assign a price to a derivative instrument in such a way that arbitrage is not necessary, and is mostly tackled from a stochastic perspective, using models such as Black-Scholes [13].

On the other hand, portfolio management is a natural area of application for computational intelligence. The problem we are interested in is the selection of optimal portfolio–a distribution of wealth over several investment assets in order to diversify risk and obtain a maximal return for the given acceptable level of risk. Typically, the higher the value of the expected return, the higher the value of risk associated with the asset. Besides the return and the risk, other factors, such as time to maturity and transaction cost, influence the decisions of how much money to invest in each asset under the consideration. There is a clear need for the development of intelligent system techniques to analyze large financial data sets, and extract relevant information for portfolio selection (see, e.g. [14], [27]).

2 Selection of Optimal Financial Portfolio

A financial portfolio is a distribution of wealth among several investment assets such as stocks, bonds, and their derivatives. With numerous possible combinations of assets, the goal is to select the optimal portfolio, where the optimality depends on the investor's goal. The two most commonly sought goals are maximization of the return for a given acceptable level of risk and minimization of risk to obtain a predefined level of return. The Nobel prize winner Harry M. Markowitz determined in early 1950s that it is impossible to increase the return without an increase in risk [20]; thus, it is typical that higher expected return is associated with higher risk. Therefore, for a rational investor, the leitmotiv to ensure an increase in wealth, is to diversify risk by investing wealth into several assets rather than into just one asset.

However, return and risk of investment assets are not the only characteristics that need to be considered when determining the best portfolio in a given situation. Time period allocated to accomplish a defined goal, time to maturity of assets, transaction cost for transferring wealth from an asset to another, and preferable structure of portfolio (e.g. risk-averse individuals would prefer investing money in low-risk government bonds and just one high-risk asset, while risk-prone individuals prefer investing in several high-risk assets and one or two low-risk assets) are just a few of numerous features that influence the selection of optimal portfolio.

Moreover, various constraints must be satisfied by the optimal portfolio. Some of the standard constraints are non-negativity of weights (i.e. amount of money allocated to each asset), the amount of money available to an investor and (often) the requirement that exactly all money is invested, maximal level of risk acceptable, and minimal return required.

Therefore, the simplest and the most natural way to represent the problem of optimal financial portfolio selection is as a constrained optimization problem. The objective is to maximize or minimize an objective function (usually maximization of return or minimization of risk) subject to constraints. The objective function and the constraints are usually not simple functions. They often depend on more than one characteristic of each asset, and these characteristics are usually combined by using functions that are much more complex than linear or quadratic functions. Thus, to find the solution to this optimization problem, general constraint solving techniques are usually not adequate, so more complex (often heuristic) techniques are needed.

3 Existing Approaches to Portfolio Selection Problem

The problem of selecting the optimal distribution of wealth among numerous assets have been approached using various techniques including return-based strategies, methods involving stochastic processes, and heuristic intelligent systems techniques. Return-based strategies are the simplest methods to optimal portfolio selection which aim at maximizing the return of portfolio. They usually do not accurately represent the real-life situation as the risk and other characteristics of assets are not considered in this approach, but they are easily solvable by using the simple linear programming techniques to find the optimal portfolio. Methods involving stochastic processes focus on predicting the behavior of assets in future (i.e. the return and the risk of the assets), which is a very important part in making the decisions of where to invest money. However, these techniques consider each asset independently and do not provide tools to determine the optimal distribution of wealth among multiple assets. Thus, even though stochastic methods are important in portfolio selection to predict the behavior of assets, they cannot be used for portfolio selection without integration into other methods. Finally, heuristic intelligent systems techniques, such as machine learning and evolutionary techniques, find a solution that is not necessarily the optimal portfolio but is close to the optimal solution. These algorithms are useful when linear programming and other simple methods are not applicable due to complexity of the objective function and the constraints. Commonly used intelligent system techniques that assist selection of optimal portfolio are support vector machines [7], neural networks ([1], [3], [18], [29], [30]), rule-based expert systems ([5], [23], [25]), and genetic algorithms ([16], [17], [28]). Their applications include prediction of the behavior of assets, selection of the best asset(s) among numerous assets in market, and selection of optimal or semi-optimal distribution of wealth among several markets. In this paper, we focus on heuristic intelligent systems techniques as they are the most complete representation of real-life situations.

Support vector machine (SVM), which is one of the most commonly used classification techniques (see e.g., [26]) is used to assist the optimal portfolio selection. In general applications, SVM classifies data in one of two groups based on training sets of data. For the portfolio selection problem, an algorithm using support vector machines was designed [7] to assist the selection of good performing assets. The algorithm using return on capital, profitability, leverage, investment, growth, short term liquidity, return on investment, and risk classifies stocks into one of two classes: the stocks with exceptional high returns and the stocks with unexceptional returns. This classification suggests which stocks should be included in a portfolio.

Another technique that has found successful application in determining the most profitable assets is the use of neural network (NN). A neural network is designed to imitate the actions of human neural system, which consists of neurons and axons (the links between neurons, which are responsible for updating data in neurons). Similarly, a neural network consists of nodes and directed links between nodes (see e.g., [26]), which carry information necessary for revision of values in nodes. NN is based on the ability to learn from training data sets in order to perform accurately on real data.

Neural networks have found several applications in portfolio management including the prediction of the behavior of investment assets and the optimization of the distribution of wealth among assets. The ability of neural network to learn from examples was used in [1] to predict the behavior of three classes of assets: stocks, bonds, and money markets. NN was trained to determine which class would outperform the other two classes based on factors such as earnings, price per earning ratio, interest rate, and inflation. The portfolio was built by investing equal amount of money in all the assets that belong to the class that was predicted to perform the best.

Another example of forecasting ability of a neural network was tested in [30], where the forecasting is based on the previous state of the asset, external influences, and the error made in previous iteration. In the other words, the difference between the predicted output and the observed output is fed back into the network as a correcting factor for future predictions. Further, the optimal distribution of wealth is determined by calculating the excess return of one asset over another based on the assets' behavior predicted by the neural network. The asset allocation step of this algorithm was improved in [29] by incorporating the risk of an asset through calculating risk-adjusted excess return of one asset over another asset.

Two other applications of neural networks for the selection of optimal portfolio are presented in literature. The optimality of portfolio in [18] is determined by risk minimization, where the risk is defined as the deviation of the predicted risk from the actual risk, while [3] presents a neural network model for wealth distribution that needs to satisfy some predefined preferred characteristics of portfolio.

Even though support vector machines and neural networks showed good results when applied to portfolio selection, other intelligent techniques have been suggested as well to optimize the distribution of wealth among assets. Rule-based expert system is one of these techniques. It simulates the decision making ability of a human expert in a field of interest (see e.g., [22]). The system is designed to allow "communication" between a client and itself through a user-interface, and then infer the optimal solution to the client's problem through the use of the inference engine, which manipulates the knowledge base containing the facts and rules of the subject at hand.

A rule-based expert system has firstly been suggested for application in portfolio selection in [5], where a single expert interviews the investor, infers the goals of the investor, and determines the optimal distribution of wealth. The interview consists of a predefined set of questions, which help the inference of the client's needs and preferences. The optimal portfolio is calculated based on the inferences and the knowledge base that contains data about assets. A more complex system was proposed in [25] and implemented in [23], where a multi-agent system for portfolio monitoring utilizes several agents to perform different tasks. Agents are trained on example data sets to perform one task, which might be interviewing the client, finding relevant information in online news, calculating portfolio risk, or suggesting to the investor the need for modification of the portfolio.

The only intelligent system technique that is not based on training data sets is genetic algorithm (GA). This optimization method imitates biological process of natural survival of the fittest individuals in a population (see e.g., [11]). Each individual is characterized by a sequence of genes, which constitute a chromosome. The fittest individuals are selected for mating. Through exchange of chromosomal material between selected pairs and through mutations of genes, the new generation is produced. Genetic algorithm simulates the three steps of natural evolutions process: selection, crossover, and mutation.

As a computational intelligence technique, genetic algorithms have found different applications in portfolio management. A portfolio is usually represented by an individual in the population, investment assets are represented by genes, and weight allocated in a particular asset is represented by the value of the gene associated with this asset.

The first application of genetic algorithm that assists in portfolio optimization was presented in [28], where a genetic algorithm was developed to select the highest performing asset among thousands of available assets. The performance in this case was defined purely by the return of assets, which is measured by the return on capital employed, price per earning ratio, earning per share, and liquidity ratio. This idea was further utilized in the first of two stages is a GA [16] that finds the optimal portfolio. The first step selects a predefined number of highest performing assets to proceed to the second stage, while the second stage utilizes another GA to distribute the wealth among selected assets. This stage of the algorithm takes into consideration both the return and the risk of the selected assets and the tradeoff between the two. With slight modifications in selection, crossover and mutation stages of the genetic algorithms, a similar two-stage process was presented in [17] to determine the optimal distribution of wealth.

4 Drawbacks of the existing approaches

We have presented several attempts to use intelligence systems in selection of optimal portfolio. Support vector machines, neural networks, rule-based expert systems, and genetic algorithms have found applications in portfolio management. However, with the exception of genetic algorithm, all other methods rely on the ability to learn from examples and make predictions based on the learned "rules". This could lead to overfitting of the parameters to a specific type of data or a specific sample, and thus not be applicable to other situations.

Moreover, all of the presented approaches assume that criteria characterizing each of the assets are mutually independent. However, in reality, it is not true. Higher return usually involves higher risk as the assets of high risk need to attract investors by offering higher return. Longer time to maturity is usually associated with higher risk since there is longer time for a company (i.e. asset) to fall apart and not be able to pay the promised amount. However, these and similar dependence are not considered in the currently used computational techniques for portfolio management.

In addition to neglecting dependence among criteria, the existing approaches do not take into consideration the dependence among assets. In real life, companies depend on each other and are not completely self-regulating. For example, if an airline company goes broke, a fuel company that was supporting this airline company will suffer as well. Therefore, these two companies are dependent and investing wealth in both of them does not diversify risk completely as it originally seems to be the case.

Furthermore, the return, the risk, and other characteristics of an asset are assumed to be precisely known for each asset in consideration. In reality, the best we can do is to predict the future return and risk, but there is no guarantee that the predictions are always correct. However, all the presented techniques rely on precise knowledge of these values. Moreover, the current approaches assume that an investor can precisely determine his/her goal. For example, it is assumed that an investor can always determine exactly how much return he/she wants. However, if an investor requests 10% return, many times, the investor might be satisfied with 9.9% return if the risk for obtaining this value is much smaller than the risk to obtain 10% return. For this reason, we can not expect an investor to give us a precise value of return he/she would like to obtain, but rather an interval of expected return.

To face the drawbacks of the presented approaches, we propose to use multi-criteria decision making and fuzzy integration over intervals to solve the problem of portfolio optimization. Therefore, we briefly review the basics of multi-criteria decision making, fuzzy integration, and intervals, and suggest why these techniques are adequate tools for solving problems involving portfolio management.

5 Multi-criteria decision making

A multi-criteria decision making (MCDM) problem seeks the optimal choice among a (finite) set of alternatives. It can formally be defined as a triple $(X, I, (\succeq_i)_{i\in I})$ where

- $X \subset X_1 \times \cdots \times X_n$ is the set of alternatives with each set X_i representing a set of values of the attribute i.
- I is the (finite) set of criteria (or attributes).
- $\forall i \in I$, $\succeq_i$ is a preference relation (a weak order, i.e., a complete transitive binary relation) over X_i.

The next task is to "combine" values of all criteria of an alternative into a global value for the alternative such that the final order of the alternatives is in agreement with the decision maker's preferences within each criterion. The most common way to construct a global preference is by using utility function for each attribute to reflect partial preferences of a decision-maker, and then combine these monodimensional utilities into a global utility function using an aggregation operator. The existence of monodimensional utility functions, $u_i : X_i \to \boldsymbol{R}$ such that for all $x_i, y_i \in X_i$, $u_i(x_i) \geq u_i(y_i)$ if and only if $x_i \succeq_i y_i$, is guaranteed under relatively loose hypotheses by the work presented in [15].

Numerous aggregation operators could be used to combine monodimensional utilities into a single number that represents the value of an alternative. Two simple approaches that correspond to optimistic and pessimistic behavior of the decision maker are maximax and maximin strategies, respectively, assuming that the goal of a decision-maker is to maximize the utility. The maximax method compares the utilities of all criteria of an alternative and chooses the highes utility value to represent the global utility of the alternative. This approach reflects the optimistic behavior of the decision-maker since he/she is concerned only with the criterion that has the highest utility value for the given alternative. On the contrary, the maximin method reflects the pessimistic behavior of the decision-maker as the decision-maker is concerned only with the criterion that could result in the worst value.

Since most of the time in real life, we do not behave completely optimistically nor pessimistically, we need to allow for a position of a decision-maker to be between these extremes. Various aggregation operators have been used for this purpose. The simplest and most natural of them is a weighted sum approach, in which the decision-maker is asked to provide weights, w_i, that reflect the importance of each criterion. Thus, the global utility of alternative $\mathbf{x} = (x_1, \ldots, x_n) \in X$ is given by $u(\mathbf{x}) = \sum_{i=1}^{n} w_i u_i(x_i)$. The best alternative is the one that maximizes this value. Even though this approach is attractive due to its low complexity, it can be shown that using an additive aggregation operator, such as weighted sum, is equivalent to assuming that all the attributes are independent [21]. In practice, this is usually not realistic and therefore, we need to turn to non-additive approaches, that is to aggregation operators that are not linear combinations of partial preferences.

Before approaching non-additive methods, we give the definition of a non-additive measure, a tool for building non-additive aggregation operators.

Definition 5.1. (Non-additive measure) *Let I be the set of attributes and $\mathcal{P}(I)$ the power set of I. A set function $\mu : \mathcal{P}(I) \to [0, 1]$ is called a non-additive measure (or a fuzzy measure) if it satisfies the following three axioms:*

(1) $\mu(\emptyset) = 0$: the empty set has no importance.

(2) $\mu(I) = 1$: the maximal set has maximal importance.

(3) $\mu(B) \leq \mu(C)$ if $B,\ C \subset \mathcal{P}(I)$ and $B \subset C$: a new criterion added cannot make the importance of a coalition (a set of criteria) diminish.

Of course, any probability measure is also a non-additive measure. Therefore non-additive measure theory is an extension of traditional measure theory. Moreover, a notion of integral can also be defined over such measures.

A non-additive integral, such as the Choquet integral [4], is a type of aggregation operator that can model the behavior of a decision maker. The decision-maker provides a set of values of importance, this set being the values of the non-additive measure on which the non-additive integral is computed from. Formally, The Choquet integral is defined as follows:

Definition 5.2. (Choquet integral) *Let μ be a non-additive measure on $(I, \mathcal{P}(I))$ and an application $f : I \to \boldsymbol{R}^+$. The Choquet integral of f w.r.t. μ is defined by:*

$$(C) \int_I f d\mu = \sum_{i=1}^{n} (f(\sigma(i)) - f(\sigma(i-1)))\mu(A_{(i)}),$$

where σ is a permutation of the indices in order to have $f(\sigma(1)) \leq \cdots \leq f(\sigma(n))$, $A_{(i)} = \{\sigma(i), \ldots, \sigma(n)\}$, and $f(\sigma(0)) = 0$, by convention.

It can be shown that many aggregation operators can be represented by Choquet integrals with respect to some fuzzy measure. Although the Choquet integral is well suited for quantitative measurements, it has a major drawback. The decision maker needs to input a value of importance of each subset of attributes, which leads to an exponential complexity and is therefore intractable. However, we can overcome intractability by using 2-additive measure to lower the complexity to a $O(n^2)$ (as shown in [2]) and still get accurate results.

Before giving the definition of a 2-additive measure, we need to define notion of interaction indices of orders 1 and 2. The importance of an attribute (or the interaction index of degree 1) is best described as the value this attribute brings to each coalition it does not belong to. It is given by the Shapley value [24]:

Definition 5.3. (Shapley value) *Let μ be a non-additive measure over I. The Shapley value of index i is defined by:*

$$v(i) = \sum_{B \subset I \setminus \{i\}} \gamma_I(B)[\mu(B \cup \{i\}) - \mu(B)]$$

with

$$\gamma_I(B) = \frac{(|I| - |B| - 1)! \cdot |B|!}{|I|!}$$

and $|B|$ denoting the cardinal of B.

While the Shapley value gives the importance of a single attribute to the entire set of attributes, the interaction index of degree 2 represents the interaction between two attributes, and is defined by ([6], [10]):

Definition 5.4. (Interaction index of degree 2) *Let μ be a non-additive measure over I. The interaction index between i and j is defined by:*

$$I(i,j) = \sum_{B \subset I \setminus \{i,j\}} (\xi_I(B) \cdot (\mu(B \cup \{i,j\}) - \mu(B \cup \{i\}) - \mu(B \cup \{j\}) + \mu(B))$$

with $\xi_I(B) = \frac{(|I|-|B|-2)! \cdot |B|!}{(|I|-1)!}$.

The interaction indices belong to the interval $[-1, +1]$ and

- $I(i,j) > 0$ if the attributes i and j are complementary;
- $I(i,j) < 0$ if the attributes i and j are redundant;
- $I(i,j) = 0$ if the attributes i and j are independent.

Even though we can define interaction indices of any order, defining the importance of attributes and the interaction indices between each pair of attributes is generally enough in multi-criteria decision making problems. Thus, 2-additive measures represent a feasible and accurate tool in this setting. The formal definition of 2-additive measure follows [6]:

Definition 5.5. (2-additive measure) *A non-additive measure μ is called 2-additive if all its interaction indices of order equal to or larger than 3 are null and at least one interaction index of degree two is not null.*

We can also show [9] that the Shapley values and the interaction indices of order two offer us an elegant way to represent a Choquet integral. Therefore, in a decision-making problem, we can ask the decision maker to give the Shapley values, I_i, and the interaction indices (of order two), I_{ij}, and then use the Choquet integral with respect to a 2-additive measure, μ, to obtain the aggregation operator:

$$(C)\int_I f d\mu = \sum_{I_{ij}>0} (f(i) \wedge f(j)) I_{ij} + \sum_{I_{ij}<0} (f(i) \vee f(j))|I_{ij}| + \sum_{i=1}^{n} f(i)(I_i - \frac{1}{2}\sum_{j \neq i} |I_{ij}|).$$

This form of the Choquet integral is practical approach to many circumstances, one of them being the selection of optimal portfolio. Since the Choquet integral with respect to a 2-additive measure takes care of dependence among the criteria, this approach could be used to overcome some drawbacks of the existing methods to portfolio selection. Neglecting dependence among criteria of each asset could be resolved by setting a multi-criteria decision making environment to determine the value of each asset based on the values of its return, risk, time to maturity and other criteria considered when making the decision. Further, a multi-criteria decision making setting could be created to account for interdependence among assets, so that wealth is not invested (with the purpose of diversifying risk) in two assets that will fail at the same time because of their mutual dependence.

Even though multi-criteria decision making setting with fuzzy integration with respect to 2-additive measures suits well the problem of optimal portfolio selection, it is not without drawbacks in its original form. The values of return and risk of each asset are predicted rather than actual values, and sometimes it is not as easy to predict these values correctly. Furthermore, it is not easy for a decision-maker to give precise value for the importance of each criterion (i.e. return, risk, etc.). A decision-maker wanting 10% return might be happy with 9.8% return if it can be accomplished in half-time or with half-risk. Therefore, it is more appropriate to represent the importance and interaction indices in form of a range of values rather than as a single value. The Choquet integral could easily be evaluated using interval arithmetics. For a complete presentation of interval arithmetic, we refer the reader to [12].

Moreover, there are often constraints imposed on the solution of an optimal portfolio. The Choquet integral could be easily modified to take into consideration these constraints. Each constraint could be represented as a penalty function whose output value is subtracted from (in the case of maximizing the Choquet integral) or added to (in the case of minimizing the Choquet integral) the value of the Choquet integral. Finding a meaningful penalty function is usually not an easy task; however, since it is easy to calculate the maximal value of the Choquet integral (with respect to 2-additive measure) for the problem of optimal portfolio selection, it is easy to define constant penalties for not-satisfied constraints. Thus, the Choquet integral with respect to 2-additive measure evaluated over intervals provides accurate and feasible tools to portfolio management.

6 Justification for the use of the Fuzzy Approach

Numerous techniques have been used to solve the problem of optimal portfolio selection. However, they all face some drawbacks including overfitting to training datasets, neglecting the dependencies among the characteristics of an asset and dependencies among assets in a market, and assuming that all data are precise. The new method based on multi-criteria decision making and fuzzy integration takes care of these drawbacks.

The Choquet integral is an aggregation operator that combines numerous criteria into one value. This method does not rely on any training data and therefore is not prone to overfitting to any data samples. Thus, once developed, this method could be used in any market, which contrasts support vector machines, neural networks, and rule-based expert systems approaches, which are prone to overfitting to training datasets and thus could only be applied in the market in which they were trained. Thus, the method based on the Choquet integral could be applied in a larger domain of markets.

Moreover, the Choquet integral is an integral based on non-additive approaches, and thus is able to represent dependencies among characteristics of an asset and among multiple assets. In contrast to other intelligent system techniques, which are not able to take into consideration these dependencies, the fuzzy approach does not naively assumes that all criteria and all attributes are independent. Thus, it is able to represent more precisely market relations and make sure that risk is indeed diversified by investing money in multiple assets.

Finally, while all the current methods ask the investor to provide the precise goals (e.g., the exact amount of return required), the new approach allows an investor to enter an interval of values for the desired goal. Since there is a tradeoff between return and risk, as well as between other characteristics of an asset, it is natural that an investor cannot precisely determine the return he/she would like to receive. It is much more reasonable that an investor could determine a desired return and an minimum acceptable level of return (i.e., a lower bound of the desired return), and therefore representing the goal as an interval is much more reasonable than representing the goal as a single number.

7 Initial Results

The methods was tested on the data from Shanghai market in the period 2000-2007 and the results were compared to three commonly used benchmark portfolios [19]. The optimal portfolio was built using the fuzzy integration approach and the return of this portfolio was compared to the return of the benchmark portfolios. The new approach outperformed (i.e., yield a higher return) all the benchmark portfolios in almost every instance. It was shown that at the significance level 0f 98%, the fuzzy integration approach yield a higher return than any of the three benchmark portfolios.

8 Conclusion

We have seen that finance is an area that is well-suited to computational intelligent approaches. Support vector machines, neural networks, rule-based expert systems, and genetic algorithms have been used in selection of optimal portfolio, one of the areas of finance that heavily relies on the computational techniques. We have suggested yet another technique to select an optimal portfolio. We have shown that a utility-based approach to decision making offers a natural and logical framework to optimize portfolio selection, and have shown how the Choquet integral (which generalizes a large class of aggregation operators in multi-criteria decision making) and interval computation can be used to solve such problems, and allow us to deal with both uncertain and imprecise data. Moreover, our approach addresses some of the issues pertaining to other computational technique approaches, such as overfitting of parameters, description of the dependencies between characteristics of an asset, representing dependencies among assets, etc.

Finally, although we have focused on portfolio management problems, it is quite possible to use similar computational technique approaches to other problems in finance such as pricing problems that have currently be solved using stochastic differential equations and stochastic integration approaches. This problem remains the part of the future work.

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