# Using Preference Constraints to Solve Multi-Criteria Decision Making Problems*


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**Abstract**

Selecting the best option among a set of alternatives is not always a straightforward process. Usually the alternatives are characterized by several attributes which all need to be considered to some extent when making the final decision. Moreover, each attribute can take several values that could be ranked based on the decision maker's preferences. Of course, we would like to choose the alternative that contains all the highest ranked attributes, but in reality, such an alternative usually does not exist. For example, when buying a car, we would like to buy the car that is the cheapest one and that has the highest safety rating, but the cheapest car usually does not have a high safety rating. The typical approach to solve this problem is to consider every existing combination of attributes and try to rank all the combinations. However, the problem quickly becomes intractable as the number of possible combinations grows exponentially when the number of attributes increases. To tackle this problem, we propose to reduce the search space (i.e., the set of all possible combinations of attributes' values) by discarding the combinations that certainly do not match the decision maker's preferable choice. Our method relies on building preference constraints and on using standard techniques to solve problems with constraints.



**Keywords:** multi-criteria decision making, preference, constraints, preference constraints, utility, Choquet integrals
**AMS subject classifications:** 90B50, 91B06, 91B08, 91B10, 91B16

*Submitted: January 19, 2009; Revised: January 11, 2010; Accepted: February 1, 2010.

# 1 Introduction

Many problems can be formulated in terms of decision making. Some decisions are relatively straightforward. However, other problems may require more strategizing such as in the case of decision pervaded by uncertainty (decision under uncertainty), or decision over multiple dimensions (multi-criteria decision making). The first difficulty faced with different criteria used to make the decisions is the domain of values that each criterion can take. Some criteria take qualitative values while others take quantitative values that might be discrete or continuous. For example, when buying a car, we might consider its color, the safety rating, and the price. The color could be expressed using qualitative values (e.g., {red, black, grey}); the safety rating could be expressed using discrete integers representing the number of stars rating a car's safety (e.g., {1,2,3,4,5}); and the price could be expressed in terms of a continuous variable taking values within an interval (e.g., $[11000, 53000]$).

Furthermore, we usually have preferences within each criterion in terms of the values we would rather choose compared to other possible values of the criterion. For example, we prefer a higher safety rating to a lower safety rating and a lower price to a higher price. However, only in miracle situations, an alternative that we can choose is characterized by the best value for each criterion. In most cases, we have to accept some tradeoffs between criteria. Moreover, we do not want to choose an alternative that is characterized by the worst values for each criterion. Thus, we need to narrow the search space from all possible values that criteria can take to a smaller space that contains the best choice (alternative) based on the preferences of an individual.

To solve this problem, we propose a method that is based on our belief that a decision maker is able to express the preference of a criterion over another criterion by means of how much of a criterion he/she would sacrifice in order to obtain a higher value of the other criterion. For example, an individual knows how much more he/she is willing to pay for a car that has one more star in safety rating. Furthermore, we propose a method to utilize these individual's preferences in order to narrow the search space.

Before going into detailed description of our proposed method, we review the basics of multi-criteria decision making problems and traditional techniques for solving these type of problems along with their advantages and drawbacks.

# 2 Multi-criteria Decision Making Problem: A Review

A multi-criteria decision-making (MCDM) problem compares multidimensional alternatives to select the optimal one. As a simple illustration of the problem, we will consider the problem of selecting the best car to buy. A MCDM problem can formally be defined as a triple $(X, I, (\succeq_i)_{i\in I})$ where

- $X \subset X_1 \times \cdots \times X_n$ is the set of alternatives with each set $X_i$ representing a set of values of the attribute $i$.
  e.g., $X_1 = \text{color} = \{\text{red}, \text{black}, \text{grey}\}$,
  $X_2 = \text{safety rating} = \{1, 2, 3, 4, 5\}$, and
  $X_3 = \text{price} = [11000, 53000]$.
- $I$ is the (finite) set of criteria (or attributes), each of them containing a finite set of possible values.

e.g., $I = \{\text{color, safety rating, price}\}$ with the possible values for each element of $I$ described by the sets $X_1, X_2$, and $X_3$ above.

- $\forall i \in I$, $\succeq_i$ is a preference relation (a weak order) over $X_i$.
  e.g., color: red $\succeq_1$ black $\succeq_1$ grey,
  safety rating: $5 \succeq_2 4 \succeq_2 3 \succeq_2 2 \succeq_2 1$, and
  price: $11000 \succeq_3 \ldots \succeq_3 53000$.

The first challenge of multi-criteria decision making problem is to "combine" the values of all criteria into a global value for the alternative such that the final order of the alternatives is in the agreement with the decision-maker's partial preferences.

# 3 Traditional Techniques to Solve Multi-Criteria Decision Making Problems

The natural way to construct a global preference is by using a utility function for each criterion to reflect partial preferences of a decision-maker, and then combine these monodimensional utilities into a global utility function using an aggregation operator. The utility functions $u_i : X_i \to \boldsymbol{R}$ such that

$$\forall x_i, y_i \in X_i, u_i(x_i) \geq u_i(y_i) \text{ if and only if } x_i \succeq_i y_i$$

map the values of all attributes onto a common scale. The existence of monodimensional utility functions is guaranteed under relatively loose hypotheses by the work presented in [6].

Different approaches exist to combine monodimensional utilities into a global value. One of the simplest methods is the maximax approach, which corresponds to an optimistic behavior of the decision maker. In this approach, the decision maker considers only the criterion with the highest utility and ignores all the other criteria. Similarly, the maximin approach corresponds to a pessimistic behavior since the decision maker takes into consideration only the criterion with the lowest utility value. Both approaches are very simple but usually do not realistically represent any behavior we exhibit in our lives unless in critical situations where purely optimistic or pessimistic behavior is necessary.

Generally, we need to consider more complex aggregation operators that take into consideration all attributes. The simplest and most natural of them is a weighted sum approach, in which the decision-maker is asked to provide weights, $w_i$, that reflect the importance of each criterion. Thus, the global utility of alternative $\mathbf{x} = (x_1, \ldots, x_n) \in X$ is given by

$$u(\mathbf{x}) = \sum_{i=1}^{n} w_i u_i(x_i).$$

The best alternative is the one that maximizes the value of $u$. Even though this approach is attractive due to its low complexity, it can be shown that using an additive aggregation operator, such as the weighted sum, is equivalent to assuming that all the attributes are independent [8]. In practice, this is usually not the case and therefore, we need to turn to non-additive approaches, that is, to aggregation operators that are not linear combinations of partial preferences.

The basic idea of non-additive (or fuzzy) approaches is the integration over fuzzy measures, where a fuzzy measure is defined as follows:

**Definition 1. (Fuzzy measure)** *Let $I$ be the set of attributes and $\mathcal{P}(I)$ the power set of $I$. A set function $\mu : \mathcal{P}(I) \to [0,1]$ is called a fuzzy measure (or a non-additive measure) if it satisfies the following three axioms:*

*(1) $\mu(\emptyset) = 0$ : the empty set has no importance.*

*(2) $\mu(I) = 1$ : the maximal set has maximal importance.*

*(3) $\mu(B) \leq \mu(C)$ if $B,\ C \subset \mathcal{P}(I)$ and $B \subset C$: a new criterion added cannot make the importance of a coalition (a set of criteria) diminish.*

A non-additive integral, such as the Choquet integral [2], is a type of a general averaging operator that can model the behavior of a decision maker. The decision-maker provides a set of values of importance, this set being the values of the non-additive measure on which the non-additive integral is computed from. Formally, The Choquet integral is defined as follows:

**Definition 2. (Choquet integral)** *Let $\mu$ be a non-additive measure on $(I, \mathcal{P}(I))$ and an application $f : I \to \boldsymbol{R}^+$. The Choquet integral of $f$ w.r.t. $\mu$ is defined by:*

$$(C)\int_I f d\mu = \sum_{i=1}^{n} (f(\sigma(i)) - f(\sigma(i-1)))\mu(A_{(i)}),$$

*where $\sigma$ is a permutation of the indices in order to have $f(\sigma(1)) \leq \cdots \leq f(\sigma(n))$, $A_{(i)} = \{\sigma(i), \ldots, \sigma(n)\}$, and $f(\sigma(0)) = 0$, by convention.*

Calculation of the Choquet integral is a practically infeasible process if the number of attributes is high since the decision maker is required to input a value for each subset of attributes, which is an exponential number of values ($O(2^n)$) with respect to the number of attributes. The speed could be improved by using the Choquet integral with respect to a 2-additive measure, which has quadratic complexity in terms of the number of criteria considered. Since the Choquet integral considers dependence among criteria, it seems to be a reasonable approach to multi-criteria decision making problems. However, the complexity of calculations of the Choquet integral and other non-additive measures ranges from quadratic to exponential.

Even though quadratic complexity is considered to be fast, finding an optimal value of the Choquet integral could be a lengthy process. Currently, the Choquet integral is optimized by using heuristic techniques such as genetic algorithm ([7],[10]) or branch and bound method [3]. Genetic algorithm explores the entire search space, which is defined as a cross product of intervals describing values of each criterion, in order to find the optimal value of the Choquet integral. Genetic algorithm is an iterative process that ends when a convergence criterion or a predefined number of iterations is reached (for a full description of a genetic algorithm, see for example [9]). If the genetic algorithm is completed without reaching the convergence criterion, it implies that the optimal solution is probably not reached. Thus, the ideal end condition of genetic algorithm is that the convergence criterion is reached. Larger the search space, larger number of iterations (and therefore more time) is needed to reach the convergence criterion. Similar conclusion holds if branch and bound method is used for the optimization of the Choquet integral: larger the search space, more iterations (and therefore time) are needed to reduce the search space to the point representing the optimum value. For this reason, we propose a method that increases the speed of optimizing the Choquet integral (or any other objective function) by initial quick reduction of the search space.

# 4 A Novel Approach to Solve Multi-Criteria Decision Making Problems: the Use of Preference Constraints

Contrary to common techniques for solving multi-criteria decision making problem, which consider the entire available search space in order to optimize the objective function (e.g., the Choquet integral), we first narrow the search space to a subspace according to decision maker's preferences. At the same time, we make sure that the best alternative for a particular decision-maker is still contained in the reduced space. To reduce the search space, we assume that the decision maker is able to express his/her preference of a criterion $A$ over another criterion $B$ by means of how much of the criterion $A$ he/she is willing to sacrifice in order to obtain a higher value of the criterion $B$. For example, we assume that the individual knows how much more he/she is willing to pay for a car that has one more star in safety rating. Of course, the sacrifice the individual is willing to make for increasing the safety rating from 1 to 2 stars might differ from the sacrifice to increase the safety rating from 4 to 5 stars. Thus, we allow the decision maker to input different tradeoffs at different levels of criteria's values.

Before describing how to reduce the search space, we need to create the original search space. For this matter, we first map the domain of each criterion into an interval. Moreover, for consistency purposes and simplicity of the entire process, we map domains of all criteria onto the interval [0,1], where higher preference is given to values closer to 1, and lower preference to values closer to 0. The cross product of all these unit intervals defines the original search space. Thus, the search space is the multi-dimensional cross product of intervals [0,1], but each interval [0,1] has a different interpretation in its dimension.

To reduce the search space, the decision-maker is asked to input how much he/she is willing to "pay" in one criterion to increase the value of the other criterion by 0.2. The value 0.2 is a randomly chosen value for demonstration purposes in this paper; we can decide to ask the client to input values of tradeoff for increase by 0.1 or 0.5 or any other value. However, once we decide on this value, we use this same value for each pair of criteria.

Note that for some pairs of criteria, "paying" does not necessarily mean giving more money, but rather how much a person is willing to sacrifice value of a criterion to increase the value of another criterion. Once we collect all the inputs from the decision maker, these preferences are used to build the constraints. For each pair of criteria, a constraint is built as a piecewise linear function in two-dimensional space. The level of sacrifice in the criterion $A$ to increase the value of the criterion $B$ from the value $x$ to the value $y$ is used to define the slope of the piece representing the constraint on the interval $[x, y]$ of the criterion whose value is being increased. A constraint is built for each pair of criteria. For each constraint, a two-dimensional space (defined by two criteria defining the constraint) is reduced to the subspace satisfying the constraint. Furthermore, the two-dimensional constraints are combined into an $n$-dimensional space by retaining only the parts of subspaces that satisfy all the constraints, thus reducing the original search space to a subspace that satisfies all the constraints. Next, we use standard state of-the-art techniques for solving problems with constraints ([1],[4],[5]) to further narrow the search space to subspace that contains the optimal value of the objective function.

# 5 A Simple Example

To illustrate the entire procedure of building preference constraints and reducing the search space based on these preferences, let us consider again the simplified example of buying a car, where we consider only three attributes: color, safety rating, and price. We first map all the criteria into some reasonable intervals. For example, if we consider only three colors (e.g., red, black, and grey), we can map the domain of colors into the interval [0,3]. Also let us assume that the buyer determines that grey is less preferable than black, which is less preferable than red. To interpret the interval [0,3], we say that values [0,1) represent green color, values [1,2) represent black color, and values [2,3] represent red color. Similarly, the domain of safety ratings (e.g., {1,2,3,4,5} representing the number of starts of safety rating) can be mapped into the interval [1,5], while the price of car is already expressed as the interval [11000,53000].

For uniformity reasons, we want to convert each of the intervals into the interval [0,1]. We can apply positive affine transformations to the domains of color and safety rating since they are already in the form that a higher value of the interval represents a higher preference. Thus, if we represent the value of the color of a car by $c$ and the safety rating by $s$, we would use the mappings

$$u(c) = \frac{c}{3} \text{ and}$$

$$u(s) = \frac{s-1}{4}$$

to map the intervals of the color and the safety rating, respectively, into the interval [0,1]. However, the domain of price is different in that a lower price represents a higher preference. Thus, in this case, we need to use negative linear mapping such as

$$u(p) = \frac{53000 - p}{42000},$$

where $p$ represents the price of a car. Therefore, by using simple affine transformations, we are able to map domain of each criterion onto the common scale [0,1].

Next, the decision maker is asked to input the importance value from 0-10 for each increase of 0.2 in the value of a criterion $A$ relative to another criterion $B$. Here, the value 0 means that the individual is not willing to sacrifice the criterion $B$ at all to increase the value of the criterion $A$, while 10 means that the individual is willing to sacrifice a lot for the increase in the other criterion.

For example, let us assume that the individual inputs the following values for the importance of safety rating relative to price:

- $0.0 \rightarrow 0.2 : 10$
- $0.2 \rightarrow 0.4 : 9$
- $0.4 \rightarrow 0.6 : 7$
- $0.6 \rightarrow 0.8 : 5$
- $0.8 \rightarrow 1.0 : 2$

which imply that to increase the safety rating from 0 to 0.2, the individual is willing to pay any price (denoted by a maximum sacrifice value of 10), whereas to increase the safety rating from 0.8 to 1, the individual is not willing to pay much (denoted by a low sacrifice value of 2).

Next, we sum all the values of importance and find the relative decreases in utility value of price for corresponding increases in safety rating. Since the sum of all values

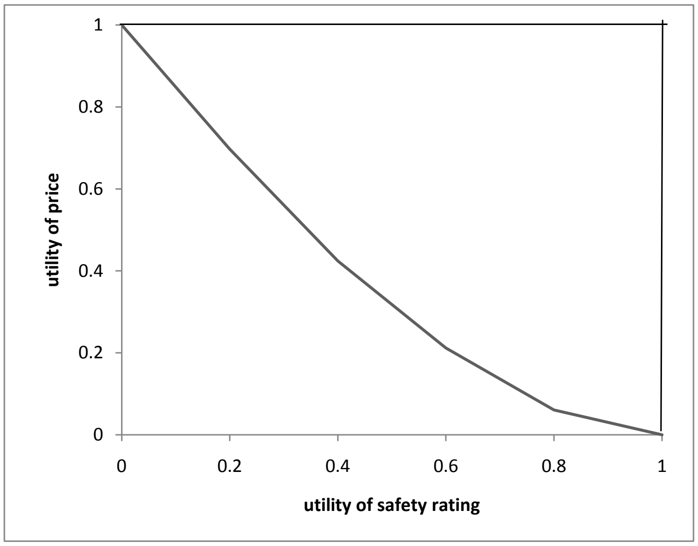


Figure 1: Tradeoff in price for increase in safety rating

given by the buyer is $10 + 9 + 7 + 5 + 2 = 33$, we conclude that increasing the safety rating from 0.0 to 0.2 is worth decreasing the utility of price by $(10/33) * 100\%$, while increasing the safety rating from 0.8 to 1.0 is worth decreasing the utility of price by $(2/33) * 100\%$. Here, we need to keep in mind that decreasing the value of utility of price actually means increasing the price of the car.

Based on the previous example, we generate the following points:

- $(0, 1)$: the initial point where no safety rating exists. Thus, the buyer is only willing to pay the minimum amount to buy a car, which is represented by the highest utility value of the preference for price (i.e., utility of price = 1).
- $\left(0.2, \frac{23}{33}\right)$: to increase the utility value of safety rating to 0.2, the buyer is willing to decrease the utility value of price by $\frac{10}{33}$; thus the resulting value of the utility value of price is $1 - \frac{10}{33} = \frac{23}{33}$.
- $\left(0.4, \frac{14}{33}\right)$: similarly to the previous case, the buyer is willing to decrease the utility of price by $\frac{9}{33}$ while increasing the utility of safety rating from 0.2 to 0.4; thus when the utility of safety rating is 0.4, the utility of price is $\frac{23}{33} - \frac{9}{33} = \frac{14}{33}$.
- $\left(0.6, \frac{7}{33}\right)$: similarly to the previous case, $\frac{14}{33} - \frac{7}{33} = \frac{7}{33}$.
- $\left(0.8, \frac{2}{33}\right)$: similarly to the previous case, $\frac{7}{33} - \frac{5}{33} = \frac{2}{33}$.
- $(1, 0)$: similarly to the previous case, $\frac{2}{33} - \frac{2}{33} = 0$. This point implies that the buyer is willing to pay maximum price (i.e., utility of price = 0) only if the car has the highest safety rating (i.e., utility of safety rating = 1).

If we connect with segments of straight lines the consecutive points generated in the example, we obtain the graph of Figure 1, which represents a preference constraint


Figure 2: Reduced search space

defined by the buyer's preferences of safety rating with respect to price of a car. The constraint is described by a continuous piecewise function. Each of its pieces is represented by a piece of straight line which is defined by the individual's inputs about preferences and willingness for tradeoffs (as described in the numerical example above).

For further use, we can either keep the linear piecewise constraint or approximate the constraint with a polynomial function that overestimates the preferred search space to ensure that the best solution is not lost during the process of approximation. For the purpose of this paper, we will keep the piecewise constraint.

Just by imposing the constraint, we have already narrowed the search space to the shaded region shown in Figure 2. This method only narrows the search space in two dimensions, but we use it as a part of narrowing the entire search space. We follow the same process to narrow the two-dimensional search spaces by using all possible pairs of criteria. In this simplified example of buying a car, we would have three constraints: safety rating vs. price, price vs. color, and safety rating vs. color.

Finally, we combine all three constraints to reduce the search space to three-dimensional cross-products of intervals that satisfy all constraints. We further use standard techniques for solving problems with constraints to reduce the search space even more in order to find the optimum value of the objective function (e.g., the Choquet integral).

Before demonstrating how to combine 2-dimensional subspaces into a 3-dimensional reduced search space (or $n$-dimensional space in general case), it is important to mention that the existing software packages that are used to solve problems with con-

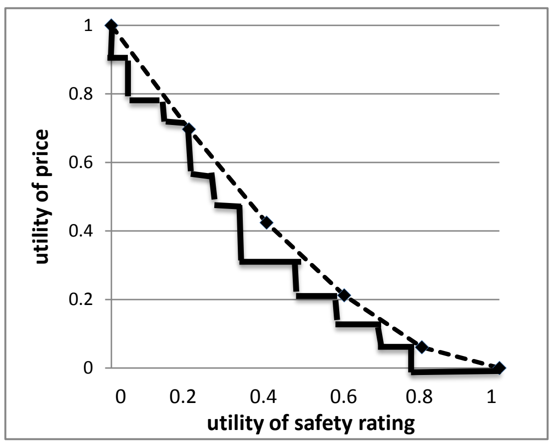


Figure 3: Constraint solver's approach

straints [5] are usually based on intervals rather than point arithmetics, and therefore the constraints are usually approximated by horizontal and vertical line segments that overestimate the search space to ensure that a possibly best solution is not lost due to approximations, which could be seen in Figure 3, where the constraint given by the buyer is represented by doted lines while the constraint solver's approximation of the search space is given by solid lines. We will demonstrate the first few steps in obtaining the constraint solver's approximation of the search space (i.e., the solid line in the Figure 3). For simplicity of demonstration, we will assume that the constraint solvers use intervals of the width 0.05 to represent data and perform calculations. However, in reality, the intervals' width is much smaller. We first consider the values on the vertical axis that correspond to the values in the interval [0,0.05] on the horizontal axis. The horizonal value of 0.05 corresponds to the value of $1 - \frac{10}{33} \cdot \frac{1}{4} = \frac{122}{132} \approx 0.9242$. Since the values of constraint solver could be represented only by the values that are multiples of 0.05 and the representation of the constraint by the constraint solver overestimates the reduced search space, the point corresponding to the value 0.05 is the largest value smaller than 0.92 that is a multiple of 0.05, which is 0.9. Thus, the point (0.05, 0.9) is one of the points represented by the constraint solver. The next point has the horizontal value of 0.1. The value of the constraint (i.e., the dashed line in the Figure 3) is $1 - \frac{10}{33} \cdot \frac{1}{2} = \frac{56}{66} \approx 0.8485$. The largest value smaller than 0.8485 that is a multiple of 0.05 is 0.8, thus the point (0.1,0.8) is the point on the constraint solver representation (i.e., the solid line in the Figure 3). All the other points are calculated similarly and the points are connected by vertical and horizontal lines to complete defining the constraints for a constraint solver.

Even though the constraints as represented by a constraint solver overestimate the preference constraints defined by a decision maker (e.g., the car buyer in our example), this approximation is still a significant improvement over the original search space as a large amount of space is discarded.

To take the full advantage of the reduces 2-dimensional search spaces, we need to combine all these subspaces into one $n$-dimensional (i.e., $n = 3$ in our example) reduced search space. To demonstrate how a constraint solver finds consistent subspaces, let us consider a subspace of the space represented in the Figure 3. The figure (as well as our calculations) suggests that the reduced 2-dimensional search space representing the pairwise values (utility of safety rating, utility of price) contains the following subspaces:

$$[0, 0.05] \times [0.9, 1],$$
$$[0.05, 0.1] \times [0.8, 1],$$

etc. Let us assume that the following subspaces are a part of the reduced search space when considering the pairwise values (utility of price, utility of color):

$$[0.75, 1] \times [0.05, 0.15],$$
$$[0.95, 1] \times [0, 0.05],$$

etc. The 3-dimensional reduced search space contains the following 3-dimensional subspaces for the triple (utility of safety rating, utility of price, utility of color):

$$[0, 0.05] \times [0.95, 1] \times [0, 0.15]$$

(because the corresponding interval for the utility of price is feasible for both intervals $[0, 0.5]$ and $[0.05, 0.15]$ of the utility of color),

$$[0, 0.05] \times [0.9, 0.95] \times [0.05, 0.15]$$

(because the corresponding interval for the utility of price is not preferable for the utility of color in the interval [0,0.05]),
and the list continues for each consecutive interval of the width 0.05 for the utility of price.

Note that a similar search space reduction method could be used to reduce the search space to the subspace that contains only existing alternatives. For example, it is common sense that the car with the highest safety rating is not the cheapest car; therefore, the point $(1, 1, x)$, where $x$ stands for any value of the utility of color, should be easily eliminated from the search space. Similarly, other approximations could be easily determined in order to reduce the search space before even beginning the optimization of the objective function.

When the search space is reduced based on preference constraints and (possibly) the constraints representing the impossible alternatives, a heuristic technique or a standard constraint solver package [5] is used in traditional way to determine an optimal alternative.

# 6 Conclusion

Many problems can be described in terms of decision making. One of the most common types of decision making is multi-criteria decision making. Different approaches exist to solve this type of problems ranging from a "quick-to-calculate" but not accurate and not realistic maximax, maximin, and weighted sum strategies to more complex to calculate and more precise non-additive approaches that involve optimizing non-differentiable functions such as the Choquet integral. To find the optimal value of a non-differentiable function heuristic techniques, such as genetic algorithm ([7],[10]), or branch and bound [3] methods are used. Both methods consider the entire search space to find the optimal value of the objective function. Larger the search space, more iterations are needed to reduce the space to a very small subspace (equivalent to a point) that contains the optimal value. In each iteration, the value of the objective function has to be calculated multiple times (e.g., usually several hundred times in an iteration of a genetic algorithm, and $2^n$ times in a branch and bound method, where $n$ is the number of criteria considered). These methods usually perform multiple iterations in the parts of the search space that certainly do not contain the optimal value of the objective function (i.e., the non-shaded region in the Figure 2).

To keep the accuracy and increase the speed of finding an optimal alternative in an MCDM problem, we presented a novel approach that represents a multi-criteria decision making problem in terms of preference constraints defined by the decision maker and uses standard techniques for solving problems with constraints to narrow the original search space while ensuring that the best solution is not lost. Even though the proposed approach has not been yet tested in a real-life applications, it is expected that this approach will speed the process of finding an optimal alternative by not wasting time to explore the parts of the search space that certainly do not satisfy a decision-maker's expectations.

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