Reviews

Applications of Reliable Scientific Computing

Рецензии

Приложения надежных научных вычислений

With this issue, we start a new section: short reviews of papers and books that describe applications of reliable scientific computing, i.e., computations with guaranteed accuracy (interval computations, etc.).

The experience of the International Workshop on Applications of Interval Computations (El Paso, TX, February 23–25, 1995) has shown that there are many unexpected application areas and results that are not widely known in the reliable computing community.

It is very difficult to trace such papers because papers of applications of reliable scientific computing are published not only in mathematical journals (that are usually covered by Mathematical Reviews, Zentralblatt fur Mathematik, etc.), but also in the journals of the corresponding application areas, that are, as a rule, not covered by traditional mathematical review journals. Moreover, reviews published in Mathematical Reviews, Zentralblatt fur Mathematik, etc., may only describe the result without explaining that interval methods (or other methods of reliable scientific computing) have been actually used. In view of this difficulty, we decided to provide the readers of RC with short reviews of application papers (something like an ongoing annotated bibliography).

We strongly believe that the information about the current applications is of interest to this community.

For the reasons expressed above, we are currently not covering all current application papers. To increase the coverage, we need your help. If you know of any recent papers devoted to applications of reliable scientific computing, please send the references to Vladik Kreinovich at `vladik@cs.utep.edu`, or by regular mail to

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If you have written your own reviews, or if you would like to write reviews, please contact Vladik as well. Authors, please send information and/or copies of your own application papers (papers in Russian and German are also welcome). Reviews should be in LaTeX, but ASCII is also acceptable.

This is a new section, and we want the reader's input about how to make it better. For example, when we have more reviews, it may make sense to divide this section into subsection devoted to different application areas. Any suggestions and recommendations will be highly welcome.

Editorial Board

Zeeman, E. C. *Catastrophe theory. Selected papers, 1972–1977.* Addison-Wesley, London, 1977.

Poston, T. and Stewart, I. N. *Catastrophe theory and its applications.* Pitman, London, San Francisco, 1978.

Guckenheimer, J. *Comments of catastrophe and chaos.* In: "Some mathematical questions in biology", IX, Lecture Notes on Mathematics in the Life Sciences, Amer. Math. Soc., Providence, RI, 1978, pp. 1-ff.

Kreinovich, V. *Letter on catastrophe theory.* Notices Amer. Math. Soc. **26** (5) (1979).

Saunders, P. T. *An introduction to catastrophe theory.* Cambridge Univ. Press, Cambridge, 1980.

Catastrophe, theory and applications. Wiley, N.Y., 1981.

Gilmore, R. *Catastrophe theory for scientists and engineers.* J. Wiley, 1981.

Arnold, V. I. *Catastrophe theory.* Springer-Verlag, Berlin, Heidelberg, N.Y., 1984.

Laws of physics are typically given in the form of a *variational principle*, i.e., each of these laws state that some characteristic S of the fields and particle coordinates, called *action*, must take *optimal* (usually, *minimal*) value. These laws can be used to predict the values of desired physical quantities based on the measurements results. The action S is usually a smooth function of its variables; therefore, e.g., in field theory, we can usually extract a system of differential equations from the variational principle and thus, predict the future values of the fields provided that we have measured their current values at all spatial points. There are infinitely many points in space, and, in reality, we cannot measure infinitely many values, so, we measure finitely many values, and use an approximate version of the variational principle (or of the resulting differential equations) to predict the values of the desired physical quantities.

In general, let us assume that we measure the quantities $m_1, \ldots, m_n$, and we are interested in the values of the quantities $x_1, \ldots, x_k$. To determine x_i from m_j, we use the variational principle

$$S(m_1, \ldots, m_n, x_1, \ldots, x_k) \to \min_{x_1, \ldots, x_k}$$

for a known function S. Differentiating over x_i, we get a non-linear system of equations: $S_i(m_1, \ldots, m_n, x_1, \ldots, x_k) = 0$, $1 \le i \le k$, where S_i is a partial derivative of S w.r.t. x_i.

Since the functions S_i are smooth (differentiable), for almost all values of $m = (m_1, \ldots, m_n)$, the dependency of x_i on m_j is also smooth. Thus, in a small neighborhood of a point m_j, there exists a constant $C > 0$ such that to determine x_i with a desired accuracy $\varepsilon > 0$, we must measure m_j with an accuracy $C \cdot \varepsilon$. In terms of the number of digits: to find d digits of x_i, we must, for some appropriate c, know m_j with an accuracy of $d + c$ binary (or decimal) digits.

The dependency of x_i on m_j is, however, not always smooth: e.g., in state equations, we have *phase transitions*, in which the dependency changes in a non-smooth manner (sometimes even discontinuously). This non-smoothness drastically decreases the accuracy of the result: e.g., if the dependency is of the type $x_i = \sqrt{m_j}$ (i.e., $x_i^2 - m_j = 0$), then for $m_j \approx 0$, to find d digits of x_i, we must know $2d$ digits of m_j (in terms of accuracy: to compute x_i with an accuracy ε, we must know m_j with an accuracy $C\varepsilon^2$ for some C). For cubic roots, the situation is even worse. In general, the higher the degree of the equation that determines x_i, the worse is the accuracy. How bad can it be?

Of course, even for a single measurement result m and a single desired variable x, we can have, in principle, an equation $x^N - m = 0$ for an arbitrary large N; as N grows larger, this equation requires better and better accuracy in m to achieve the same accuracy in x. So, in principle, it can be as bad as possible.

The next natural question is: how frequent are these bad cases? Are almost all situations bad or bad situations are (in some reasonable sense) rare?

Catastrophe theory is a formalism that provides a partial answer to this question. Its basic result, first proven by R. Thom and E. C. Zeeman, states that if the number k of the unknowns is 5 or less, then for *almost all* functions S (in some reasonable topological sense) the solution $x_i(m_1, \ldots, m_n)$ of the equations $S_j = 0$ can be locally represented as a composition of three mappings:

- a *smooth* mapping $m_1, \ldots, m_n \to m'_1, \ldots, m'_n$;
- a mapping $m'_j \to x'_i$ described by the condition

$$S_a(m'_1, \ldots, m'_n, x'_1, \ldots, x'_k) \to \min$$

  where S_a is a function from a finite list of polynomial functions called *elementary catastrophes*;
- a *smooth* mapping $x'_1, \ldots, x'_k, m_1, \ldots, m_n \to x_1, \ldots, x_n$.

Thus, for *almost all* functions S, the only part of the algorithm that requires a non-linear increase in accuracy is the second one, and in this second part, the degree of the polynomial S_a (and hence, the increase in accuracy) is bounded.

This result is proven for *almost all* systems S; it might so happen that it is not true for the actual physical action functions S; luckily, however, this decomposition result is true for known physical action functions as well.

When we represent a mapping in the form of this composition, then the only part of this mapping in which we must take special care of accuracy (because linearlization estimates do not work) is the second one, in which we actually follow one of the *standard* systems S_a. For these finitely many standard systems, we can pre-compute the desired accuracy.

So far, we talked about the situations in which we already know S. In many real-life problems, however, we know that there is a variational principle, but we do not know the exact function S. In such situations, we must determine S from the experiments. Usually, to determine S, we expand S into a power series (cut after a certain power), and determine the (unknown) coefficients of this series from the results of the experiments. We have mentioned that it is computationally advantageous to find x_i using the three-mapping representation. Therefore, rather than finding S and determining the mappings, it is easier to find the mapping directly from the experiments: namely, we expand the formulas for these mapping into Taylor series, and find the unknown coefficients of these expansions directly from the experimental results.

This methodology forms of the basis of *applied catastrophe theory*. There exist many successful applications of this methodology (and even more suggestions and speculations on the possibility of other applications).

Comment. Our description of catastrophe theory is aimed at the interval computations community, and is, therefore, different from the usual expositions of this theory: Although the computational (including computational accuracy) aspects of catastrophe theory are (implicitly or explicitly) present in the papers and monographs, these aspects are usually overshadowed either by complicated mathematics, or by the description of successful applications, or (as in original papers of Thom) by philosophy.

Th. Swenson and V. Kreinovich

Marks, M. and Hammond, K. J. *A review of Allen, J. F., Kautz, H. A., Pelavin, R. N., and Tenenberg, J. D. "Reasoning about plans", Morgan Kaufmann, San Mateo, CA, 1991.* ACM SIGART (Special Interest Group in Artificial Intelligence) Bulletin **4** (2) (1993), pp. 8–11.

The book that Marks and Hammond review describes the use of intervals for planning. Marks and Hammond emphasize that the interval approach to planning (originated from the works of Allen) is based on the assumption that expert knowledge about time is *consistent*. If expert specifications turn out to be inconsistent, then we must inform the experts about this inconsistency and ask them to correct their specifications. This tedious procedure is absolutely necessary in critical situations, when every requirement is absolutely necessary (*hard*). In real life, however, many requirements are *soft* (recommendations rather than requirements), so inconsistency and the resulting impossibility to satisfy all the requirements may be quite admissible.

M. Beltran

Allen, J. F. and Ferguson, G. *Actions and events in interval temporal logic.* J. Logic Computat. **4** (5) (1994), pp. 531–579.

The authors describe an interval-based temporal logic in which usual non-temporal basic statements of the type $P(s)$ ("the property P holds for objects s") are replaced by temporal statements $P(s, [t^-, t^+])$ ("the property P holds for objects s for all moments of time $t \in [t^-, t^+]$"). The basic relation between intervals is "meets": $[t^-, t^+]$ meets $[s^-, s^+]$ iff $t^+ = s^-$. All other ordering relations between intervals can be expressed in terms of "meets": e.g., "$\mathbf{t} = [t^-, t^+]$ precedes $\mathbf{s} = [s^-, s^+]$" (meaning $t^+ < s^-$) is equivalent to

$$\exists \mathbf{u}\,(\mathbf{t} \text{ meets } \mathbf{u} \,\&\, \mathbf{u} \text{ meets } \mathbf{s}).$$

The authors show how this logic can be used to represent expert knowledge about actions, and how it can be used for prediction, planning, and explanations.

A. Provetti

Barth, W., Lieger, R., and Schindler, M. *Ray tracing general parametric surfaces using interval arithmetic.* The Visual Computer, International Journal of Computer Graphics **10** (7) (1994), pp. 363–371.

This paper describes an algorithm for ray tracing general parametric surfaces. After dividing the surface adaptively into small parts a binary tree of these parts is built. For each part a bounding volume is calculated with interval arithmetic. From linear approximations and intervals for the partial derivatives it is possible to construct parallelepipeds that adapt the orientation and shape of the surface parts very well and form very tight enclosures. Therefore an algorithm for rendering can be developed which is similar to that used with Bezier- and B-Spline-surfaces, where the bounding volumes are derived from the convex hull property. The tree of enclosures (generated once in a preprocessing step) guarantees that each ray that may hit the surface leads to an iteration on a very small surface part; this iteration can be robustly (and very fast) performed in real arithmetic.

W. Barth, R. Lieger, and M. Schindler

Atallah, M. J., Chen, D. Z., and Lee, D. T. *An optimal algorithm for shortest paths on weighted interval and circular-arc graphs, with applications.* Algorithmica **14** (1995), pp. 429–441.

How to design a traffic control scheme for a long highway of length L? We have n possible control teams; team # i can effectively control traffic on an interval $[a_i^-, a_i^+]$, and the cost of this team's use is c_i. We must select the teams so that the entire highway is covered, and the total cost is the smallest possible.

Similar problems emerge:

- in *scheduling*; e.g., if we schedule the teaching assistants to supervise a computer lab;
- in *biology*, where, e.g., we must decode the DNA in the cheapest possible way by decoding its segments $[a_i^-, a_i^+]$;
- in *VLSI design*, where we must find the cheapest set of tests that covers the entire path of the signal;
- and in many other application areas.

The traditional way to solve this problem is to design a graph whose nodes are intervals $\mathbf{a}, \mathbf{b}, \ldots$, and in which $\mathbf{a}$ and $\mathbf{b}$ are connected iff $\mathbf{a} \cap \mathbf{b} \neq \emptyset$. Such a graph is called a *(weighted) interval graph*, and the problem is reduced to finding a shortest path in this graph.

Traditional shortest path algorithms require time that is quadratic in the size n of the input. In this paper, a new *linear time* algorithm is proposed. (To be more precise, this algorithm requires linear time if we assume that the set of all endpoints $a_i^{\pm}$ is already ordered; otherwise, it takes time $O(n \log(n))$.)

In many real-life problems, we have a similar problem: we have *arcs* on a *circle*, and we must find the cheapest set of arcs that covers the entire circle. For example, in traffic control, the road may be a loop. For this problem, the authors propose a quadratic-time algorithm.

V. Kreinovich

Barth, W. *Using special arithmetic in geometric algorithms.* To appear in: "Numerical Methods and Error-Bounds", Proceedings of the IMACS-GAMM International Symposium, University of Oldenburg, Germany, AKADEMIE Verlag, Berlin, July 1995.

The application of interval arithmetic shown in this paper deals with geometric algorithms. The purpose for which we use this reliable computation is different from the usual one. It is not to find an inclusion for the final results, but it is to control the algorithm correctly, i.e. to find the right branch the algorithm has to follow from the signs of intermediate results, this means assuring reliable control. The problem arises when working with the method of Edelsbrunner and Mücke for eliminating all special cases in geometric algorithms. Of course the sign of an intermediate result (arithmetic expression) can only be determined correctly without any exception by exact arithmetic. But it is much more efficient to calculate such an expression first with interval arithmetic, and only in the few cases where the resulting interval contains zero the time consuming exact calculation has to be performed.

W. Barth

Bender, M. A., Gastaldo, M., and Morvan, M. *Parallel interval order recognition and construction of interval representations.* Theoretical Computer Science **143** (1995), pp. 73–91.

The authors propose parallel algorithms for solving the following scheduling problem: we have n tasks $t_1, \ldots, t_n$, and we have selected m pairs $(t_{f(i)}, t_{s(i)})$ of these tasks $(1 \le i \le m)$ that must overlap in time. We must *schedule* these m tasks (i.e., map each of them into a time interval) in such a way that only intervals that correspond to m given pairs of tasks overlap, and intervals corresponding to other pairs don't overlap. The proposed algorithms run in time $O(\log(n))$ on $n + m$ processors and in constant time on $O(n^2)$ processors.

V. Kreinovich

Blondel, V. *On interval polynomails with no zeros in the uni disc.* IEEE Transactions on Automatic Control **40** (3) (1995), pp. 479–480.

It is well known that a discrete-time linear system $x(k+1) = Ax(k)$ if stable iff the eigenvectors $\bar{x}$ of A (i.e., vectors for which $A\bar{x} = \lambda\bar{x}$ for some λ) lead to stable trajectories $x(k) = \lambda^k\bar{x}$, i.e., iff $\lambda^k \to 0$ when $k \to \infty$. This, in its turn, is equivalent to $|\lambda| < 1$. One way to compute eigenvalues of a given matrix A is to take into consideration that eignevalues are roots of the the characteristic polynomial $\bar{p}(\lambda) = \det(A - \lambda I)$. In terms of p, the above condition means that all roots of the polynomial $\bar{p}(\lambda) = p_0\lambda^n + p_1\lambda^{n-1} + \cdots + p_n$ must be inside the open unit disk. It is often convenient to consider a new variable $z = 1/\lambda$. To guarantee stability, this new variable must satisfy the property $|z| > 1$. This new variable is a root of the polynomial equation $p(z) = 0$, where $p(z) = \bar{p}(1/\lambda) \cdot \lambda^n = p_0 + p_1 z + \cdots + p_n z^n$. For this new polynomial, the stability condition is that $p(z)$ should have no zeros in the closed unit disk.

When we start with a system with interval uncertainty (i.e., with an interval matrix $\mathbf{A}$), we end up with a polynomial $\mathbf{p}(z) = \mathbf{p}_0 + \mathbf{p}_1 z + \cdots + \mathbf{p}_n z^n$ with interval coefficients $\mathbf{p}_i$.

In the paper under review, a new necessary condition is formulated for all roots of an interval polynomial to be outside the closed unti disk. This condition uses only two coefficients $\mathbf{p}_0$ and $\mathbf{p}_1$.

V. Kreinovich

Bustince, H. and Burillo, P. *Correlation of interval-valued intuitionistic fuzzy sets.* Fuzzy Sets and Systems 74 (2) (1995), pp. 237–244.

An expert system is a computer system that contains and uses expert knowledge. Expert statements that constitute this knowledge are often formulated in terms of natural language that do not have a precise meaning: e.g., the expertize in controlling a car can be described in terms of the rules of the type "if you are driving *fast*, and an obstacle is *very close*, break *hard*" (the italicized words are not formally defined). The problem with describing the meaning of such words to a computer is that, for some values of velocity, the expert is not 100% sure whether these values mean "fast" or not. So, to describe the meaning of the term A that characterizes some quantity q, we must, for each value of q, characterize the expert's degree of belief $\mu_A(q)$ that q satisfies the property A.

The simplest way to describe uncertainty is to characterize the expert's degree of belief in $A(q)$ by a number $\mu_A(q)$ from the interval $[0,1]$ (e.g., by asking the expert to describe his/her degree of belief d on a scale from 0 to, say, 10, and then assigning $\mu_A(q) = d/10$). The resulting function μ_A from the set Q of possible value of q to the interval $[0,1]$ is called a *fuzzy set*.

If we know the degree of belief $\mu_A(q)$ in $A(q)$, then we can estimate the degree of belief in $\neg A(q)$ as $1 - \mu_A(q)$.

A fuzzy set is not a perfect description of expert's uncertainty: e.g., it assigns 0.5 both to the case when we have exactly as many arguments in favor of $A(q)$ and in favor of $\neg A(q)$, and to the case when we do not anything about $A(q)$. To distinguish between these two situations, we can ask an expert to estimate *two* values: his/her degree of confidence $\mu_A(q)$ in $A(q)$, and degree of confidence $\mu_{\neg A}(q)$ in $\neg A(q)$. In the resulting formalism (called *intuitionistic fuzzy logic*), the degree of belief in $A(q)$ is characterized by an *interval* $[\mu_A(q), 1 - \mu_{\neg A}(q)]$ (with $\mu_A(q) \le 1 - \mu_{\neg A}(q)$).

This description is better but still not perfect because in reality, it is often difficult for an expert to pinpoint his degree of confidence very narrowly. A more realistic description of an expert's degree of confidence is an *interval* of possible values $[\mu_A^-(q), \mu_A^+(q)]$. If we only consider the degree of belief in A, then we get *interval-valued fuzzy sets*. If we consider interval degrees of confidence for both A and $\neg A$, then we get an *interval-valued intuitionistic fuzzy set*, in which the degree of belief is described (using an appropriate interval term) by a *twin*

$$[\mathbf{m}_A(q), 1 - \mathbf{m}_{\neg A}(q)]$$

where $\mathbf{m}_A(q) = [\mu_A^-(q), \mu_A^+(q)]$ and $\mathbf{m}_{\neg A}(q) = [\mu_{\neg A}^-(q), \mu_{\neg A}^+(q)]$.

An expert can formulate many rules; the more rules, the slower the resulting expert system. Hence, to speed up the expert system, we must delete the rules that are redundant in the sense that they follow from the others. In particular, if we have two rules "if A then..." and "if B then..." with different conditions A and B, then, we would like to know to what extent the properties A and B are *correlated*.

If A and B are both described by fuzzy sets, then, as a degree of correlation, we can use the degree to which A and B occur together, i.e., the degree of belief in $A\&B$. If we use the product to describe $\&$, then, for each q, we get the formula $I(A,B,q) = \mu_{A\&B}(q) = \mu_A(q) \cdot \mu_B(q)$. So, as a degree of correlation, we can take the average value of $\mu_{A\&B}(q)$ for all q: $I(A,B) = |Q|^{-1} \sum_{q \in Q} I(A,B,q)$. The properties A and B are correlated if $A \approx B$, and hence, if $I(A,B) \approx I(A,A) \approx I(B,B)$. So, we can take $K(A,B) = I(A,B)/\sqrt{I(A,A) \cdot I(B,B)}$ as the *correlation coefficient* that takes values between 0 and 1, and that is equal to 1 iff $A = B$.

If A and B are described by intuitionistic fuzzy sets, then for every q, we have an interval $[\mu_A(q), 1 - \mu_{\neg A}(q)]$ of possible values of degree of belief, and therefore, we get an *interval* of possible values of $\mu_{A\&B}(q)$:

$$\left[\mu_A(q) \cdot \mu_B(q), \left(1 - \mu_{\neg A}(q)\right) \cdot \left(1 - \mu_{\neg B}(q)\right)\right].$$

As a numerical value $I(A,B,q)$ of correlation, it is natural to take an average value from this interval, i.e., its midpoint

$$I(A,B,q) = \frac{1}{2} \cdot \left(\mu_A(q) \cdot \mu_B(q) + \left(1 - \mu_{\neg A}(q)\right) \cdot \left(1 - \mu_{\neg B}(q)\right)\right).$$

For interval-valued intuitionistic fuzzy sets, we have intervals for $\mu_A(q)$ and for $\mu_B(q)$. As a result, the above formula leads to an interval. Again, to get a numerical values, it is natural to take a midpoint

of this interval; this idea leads to the formula

$$I(A,B,q) = \frac{1}{4}\cdot\Big(\mu_A^-(q)\cdot\mu_B^-(q)+\mu_A^+(q)\cdot\mu_B^+(q) + \Big(1-\mu_{\neg A}^-(q)\Big)\cdot\Big(1-\mu_{\neg B}^-(q)\Big)+\Big(1-\mu_{\neg A}^+(q)\Big)\cdot\Big(1-\mu_{\neg B}^+(q)\Big)\Big).$$

The paper under review introduces the above definitions and describes the properties of thus defined correlation.

V. Kreinovich

Chen, S., Qiu, Z., and Song, D. *A new method for computing the upper and lower bounds on frequencies of structures with interval parameters.* Mechanics Research Communications **22** (5) (1995), pp. 431–439.

The frequency ω of a structure can be determine as a square root $\omega = \sqrt{\lambda}$ of the solution to the so-called *generalized eigenvalue problem* $Ku = \lambda Mu$, where $K = |k_{ij}|$ is a *stiffness matrix* and $M = |m_{ij}|$ is a *mass matrix*. In real life, we often know only the intervals of possible values of k_{ij} and m_{ij}; in such situations, we want to know the interval of possible values of λ.

There exist several interval methods for solving the generalized interval eigenvalue problem; these methods are mainly *algebraic*, based on the equation $Ku = \lambda Mu$. In this paper, a new method is proposed that is based on the known representation of the eigenvalue problem as an optimization problem (called *Rayleigh Quotient*): the Largest eigenvalue is equal to

$$\lambda = \max_{u\neq 0}\frac{u^T K u}{u^T M u}$$

similar formulas describe other eigenvalues (the corresponding formulas are slightly more complicated, with min max instead of max).

The method is illustrated on the example of a multi-story structure.

V. Kreinovich

Kogan, J. and Leizarowitz, A. *Frequency domain criterion for robust stability of interval time-delay systems.* Automatica **31** (3) (1995), pp. 463–469.

Stability conditions for time-delay systems are of great importance for industrial applications. Delays often occur in the transmission of information or material between different parts of a system. Transportation systems, communication systems, chemical processing systems, metallurgical processing systems, environmental systems, and power systems are examples of time-delay systems. The mathematical formulation of a time-delay system results in a system of delay-differential equations. Any mathematical model of an engineering system possesses the unavoidable inaccuracy. The existence of the inaccuracies implies that the analysis of stability and performance as well as system design, based on a nominal model only, may not be meaningful in applications.

In this paper, a new criterion is described for robust stability of interval time-delay systems.

From the authors' summary

Li, R. and Carmo, J. *On completeness of a positional interval logic with equality, overlap and subinterval relations.* Journal of the IGPL (Interest Group on Propositional and Predicate Logic) **3** (5) (1995), pp. 765–790.

A new logic for reasoning about time intervals is presented and proven to be *complete* in the standard logical sense: if a statement S is true in all possible situations, then S can be deduced from the axioms of this logic by using its deduction rules. This result can be used to design a computer system that would automatically check whether a given statement about time intervals is true or not.

V. Kreinovich

Luo, J. S., Johnson, A., and van der Bosch, P. P. J. *Delay-independent robust stability of uncertain linear systems.* Systems and Control Letters **24** (1995), pp. 33–39.

The authors consider linear systems with uncertainty and time delay that are described by the equations of the following type:

$$\dot{x}(t) = Ax(t) + B_0x(t-\tau_0) + \sum_{j=1}^{r} \beta_j B_j x(t-\tau_j).$$

Here:

- The uncertainty in a matrix A is caused by m unknown factors α_i: $A = A_0 + \alpha_1 A_1 + \cdots + \alpha_m A_m$; the matrices A_i (that describe the dependence of A on these factors) are known; the values of the factors α_i are unknown, but we know the bounds μ_i for these values: $\alpha_i \in [-\mu_i, \mu_i]$.
- The coefficients β_j at the "unwanted" delay terms are unknown; we know the bounds ν_j for these coefficients ($\beta_j \in [-\nu_j, \nu_j]$), and we know the matrices B_j that describe the influence of these delays on the system.
- The values of the delays τ_k are unknown.

Based on the available information, we want to find out whether the system is stable or not; to be more precise, whether the system is guaranteed to be stable (i.e., is *robustly stable*), or it may not be stable for some possible values of the unknown parameters α_i, β_j, and τ_k. The authors describe new sufficient criteria for robust stability of such systems.

V. Kreinovich

Majumdar, S. *Application of relational arithmetic in performance analysis of computing systems.* In: "Workshop on Interval Constraints (International Logic Programming Symposium ILPS'95)", Portland, OR, December 1995.

This paper presents a survey of the existing work in the area of interval arithmetic-based performance analysis of computing systems.

Intervals in performance analysis are required when uncertainties or variabilities exist in the workload parameters for an analytical performance model of the system. Intervals are also useful for computing upper and lower bounds on system performance.

Most conventional analytic models accept a set of single valued parameters and produce a single valued model output. Adaptation of these existing models to handle interval parameters require new techniques that use an interval arithmetic engine.

Experiences with relational interval arithmetic provided by a constraint logic programming languages in solving a number of performance analysis problems are described.

Previous publications on the applications of interval analysis to performance evaluation of computing systems include:

Majumdar, S. and Ramadoss, R. *Interval-based performance analysis of computing systems.* In: "Proc. Modeling, Analysis, and Simulation of Computer and Telecommunication Systems", January 1995, pp. 345–351.

Majumdar, S., Woodside, C. M., Neilson, J. E., and Petriu, D. C. *Robust box bounds: network performance guarantees for closed multiclass queuing networks with minimal stochastic assumptions.* In: "Proc. of Infocom'92 Conference", Florence, Italy, May 1992, pp. 2006–2016.

Majumdar, S., Woodside, C. M., Neilson, J. E., and Petriu, D. C. *Performance bounds for concurrent software with rendezvous.* Performance Evaluation **13** (4) (1991), pp. 207–236.

Majumdar, S. *Interval arithmetic for performance analysis of distributed computing systems.* In: "Proc. Canadian Conference on Electrical and Computer Engineering", Quebec City, Canada, September 1991, pp. 32.3.1–32.3.4.

Ramadoss, R. *Interval-based performance analysis of computing systems.* M. Eng. Thesis, Department of Systems and Computer Engineering, Carleton University, Ottawa, Canada, 1994.

Woodside, C. M., Majumdar, S., and Neilson, J. E. *Interval arithmetic for computing performance guarantees in client-server systems.* In: Dehne, F., Fiala, F., and Koczkodaz, W. W. (eds) "Lecture Notes in Computer Science: Advances in Computing and Information—ICCI'91", Springer-Verlag, 1991, pp. 535–546.

S. Majumdar

Mori, T. and Kokame, H. *Comments of "On the stability of discrete-time linear interval systems".* Automatica **31** (6) (1995), pp. 921–922.

The authors comment on the paper by P. Myszkorowski published in *Automatica* **34** (1994), pp. 913–914. In that paper, the author proposes a new sufficient condition for stability of discrete-time linear systems $x_{k+1} = A(k)x_k$, where for every k, components $a_{ij}(k)$ of the matrix $A(k)$ belong to the known intervals $[a_{ij}^-, a_{ij}^+]$. Myszkorowski's criterion is difficult to check. The authors show that his criterion is equivalent to the easily checkable fact that $I - B$ is an M-matrix, where I is a unit matrix, $b_{ij} = \max(|a_{jj}^-|, |a_{ij}^+|)$, and an M-matrix is a matrix with non-positive off-diagonal entries for which successive leading principal minors are all positive.

V. Kreinovich

Mori, T. and Kokame, H. *Stability criteria for interval matrices via regularity conditions.* IEICE Trans. Fundamentals **E78–A** (5) (1995), pp. 553–555.

In order to check that a given control $u = Kx$ makes a given system $\dot{x} = \mathbf{A}x + \mathbf{B}u$ with intervally uncertain coefficient matrices $\mathbf{A}$ and $\mathbf{B}$ stable, we must check whether the interval matrix $\mathbf{A} + \mathbf{B}K$ is *stable* (i.e., whether $\mathrm{Re}(\lambda) < 0$ for all its eigenvalues λ). In general, checking stability is an NP-hard problem. There exist several algorithms for checking stability (including several proposed by J. Rohn); these algorithms require, in the worst case, exponentially long time.

In this paper, the author proposes several new, easily checkable, sufficient criteria for stability of interval matrices.

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