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Parallel interval-based reasoning in medical knowledge-based system Clinaid

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The paper continues a series of papers and a monograph [33], where we have described the conceptual structures as well as the basic architecture of a knowledge-based system Clinaid.

Параллельный интервальный вывод в Clinaid — медицинской системе, основанной на знаниях

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Статья продолжает ряд работ, включая монографию [33], в которых описываются базовые концепции и основы архитектуры Clinaid — системы, основанной на знаниях

1. Introduction

In a series of papers and a monograph [33], we have described the conceptual structures as well as the basic architecture of a knowledge-based system Clinaid. Its architecture is aimed at supporting not only diagnosis but also other types of clinical activity and decision making in diverse clinical and/or hospital environments. The aim of the generic architecture of Clinaid is to support knowledge-based decision making under conditions of risk and uncertainty, both of which are present in clinical medicine. Such a system has to operate in a multi-environmental situation and make decisions within a multiplicity of contexts. The basic architecture consists of the following co-operating units (basic shell substratum):

1. Diagnostic Unit (comprized of several parallel co-operating centres).
2. Treatment Recommendation Unit.
3. Patient Clinical Record Unit.
4. Co-ordination and Planning Unit.

The majority of extant medical expert systems deal with a limited medical context, the largest domain of knowledge being just a single medical field, e.g. Internal medicine in CADUCEUS The inherent limitation of such medical expert systems is in its essence conceptual and logical· their knowledge bases and inference engines cannot mix easily knowledge from several fields without some adverse effects. CLINAID deals with this problem by introducing a multi-centre architecture in the Diagnostic Unit. This naturally leads to parallel processing. The medical data and knowledge of each medical specialist field exhibits different logical properties. This in turn leads to several kinds of many-valued logics on which the relational inference and data manipulation is based. The semantic justification of these logics is provided by a theoretical device called the *checklist paradigm* [16, 46] that was introduced by Bandler and Kohout in 1979, in order to provide semantics for fuzzy connectives. It turns out that this semantic device also gives an epistemological justification for the interval-valued inference that is used to a great advantage in CLINAID's parallel inferential system. The interval-valued representation is also used to represent the clinical data and retrieve this data using the retrieval techniques that utilize information structures carrying interval credibility weights.

As CLINAID attempts to be a comprehensive medical consultation system, its knowledge has to contain a large amount of medical expert knowledge. The Diagnostic unit of CLINAID deals with a number of body systems [37]. In this paper we shall use the *cardiovascular* body system knowledge base of CLINAID to provide concrete examples.

2. Concurrency and parallelism in clinical decision making

Although there exist a number of successful medical expert systems within a narrow specialist domain which are capable of operating in an experimental or laboratory medical environment, there is a long way to go toward the completion of a comprehensive knowledge based system that would be widely accepted in medical clinics and hospitals. One of the reasons is the fact that the extant systems do not emulate sufficiently the **inherently concurrent nature** of clinical decision making [27].

It is usually assumed that medical decision making consists only of medical diagnosis, which in turn is understood to be the identification of a disease. This is supposed to consist of simply matching the patient's symptoms and all other indicators with the list of symptoms and indicators for a subset of all known diseases and finding the best match. Yet, real live medical decision making involves mostly decisions under risk, uncertainty and often in emergencies when the time factor is crucial. The time limit in emergencies requires a *fast response*, hence real-time concurrency is essential. Uncertainty and inherent incompleteness of observed or measured patient data leads to approximation of degrees of plausibility assigned to each item of data. Practice shows, that once plausibility degrees are introduced, more *experienced clinicians* are reluctant to assign a single number to the observed sign, but rather prefer to *indicate the interval* the plausibility degree might belong to.

It has been shown [20] that medical decision making, in addition to diagnosis contains many other activities including planning. The clinician not only determines the diagnosis of a patient, but also makes a large number of other decisions, including administrative ones; trying to deal with a multitude of diverse and often conflicting requirements. In real life, all these requirements have to be considered concurrently, with different priorities attached to each concurrent activity. This is crucial in particular in emergencies and other life threatening conditions. We have structured our generic CLINAID multi-environmental and multi-context knowledge-based system architecture [1, 33] is such a way that it can satisfy these essential requirements. This, however, has led us to reconsider the problems of parallel computation, interval based knowledge representation and inference within a joint framework, that is a framework that can reconcile and unify the multitude of facets involved in this problem.

3. The Clinaid architecture

The CLINAID architecture [2, 3], as indicated above, consists of four main co-operating units, where each unit is a quite complex autonomous subsystem. The global configuration of this architecture, including the data and control flow interaction between its main parts is shown in Figure 1. The greatest degree of concurrency can be achieved in the Diagnostic Unit which is comprized of a number of parallel co-operating centres. Each centre of the Diagnostic Unit is processing its own input and may be co-ordinating several parallel threads of computation within its own structure. This parallel function is facilitated by relational representation of inference (cf. Section 5 below) which also supports interval-based reasoning.

As is apparent from Figure 1, also the other parts, namely the Treatment Recommendation Unit, Patient Clinical Record Unit and Co-ordination and Planning Unit have to interact concurrently. Each of these global units is at the same time a knowledge source and an information consumer. The operation of communicating knowledge sources is an important issue of its own standing (cf. [43]). As we are concerned in this paper mainly with the interplay of issues of parallelism with interval-based inference, we shall direct our presentation in subsequent sections towards a more detailed exposition of the multi-centre Diagnostic Unit. We shall proceed now with a more detailed specification of activities of the individual CLINAID units that is needed for better appreciation of the global functions of CLINAID and of their dynamics.

3.1. The diagnostic unit

The purpose of the unit is to assist in the diagnosis of diseases that are within its purview. During this activity it accepts the information required in a dynamic manner It is not connected to the patient but relies on requests to the user (clinician) for further information so long as it lacks sufficient information to recommend a working diagnosis. As it acquires more and more information, it restricts the set of the most likely diseases to fewer and fewer. During this activity, the unit is in potential communication with the treatment recommendation unit. This is in order to deal with possible emergencies if necessary, before the working diagnosis is reached. Finally the unit either proffers a conclusion for the user's consideration or, if it still lacks sufficient evidence, an indication is given that the unit is unable to reach a final diagnostic conclusion. If a conclusion is reached then this conclusion is communicated to the Treatment Recommendation Unit.

During the inference process, the Diagnostic Unit requires the experts' knowledge and experience which is obtained from the Knowledge Base Sometimes this unit also requires use of the patient's medical record when it works towards a diagnosis. All the signs, symptoms, the results of physical examinations, laboratory tests and the conclusion are communicated to the Patient Clinical Record Unit The occurrence of this communication depends on the context

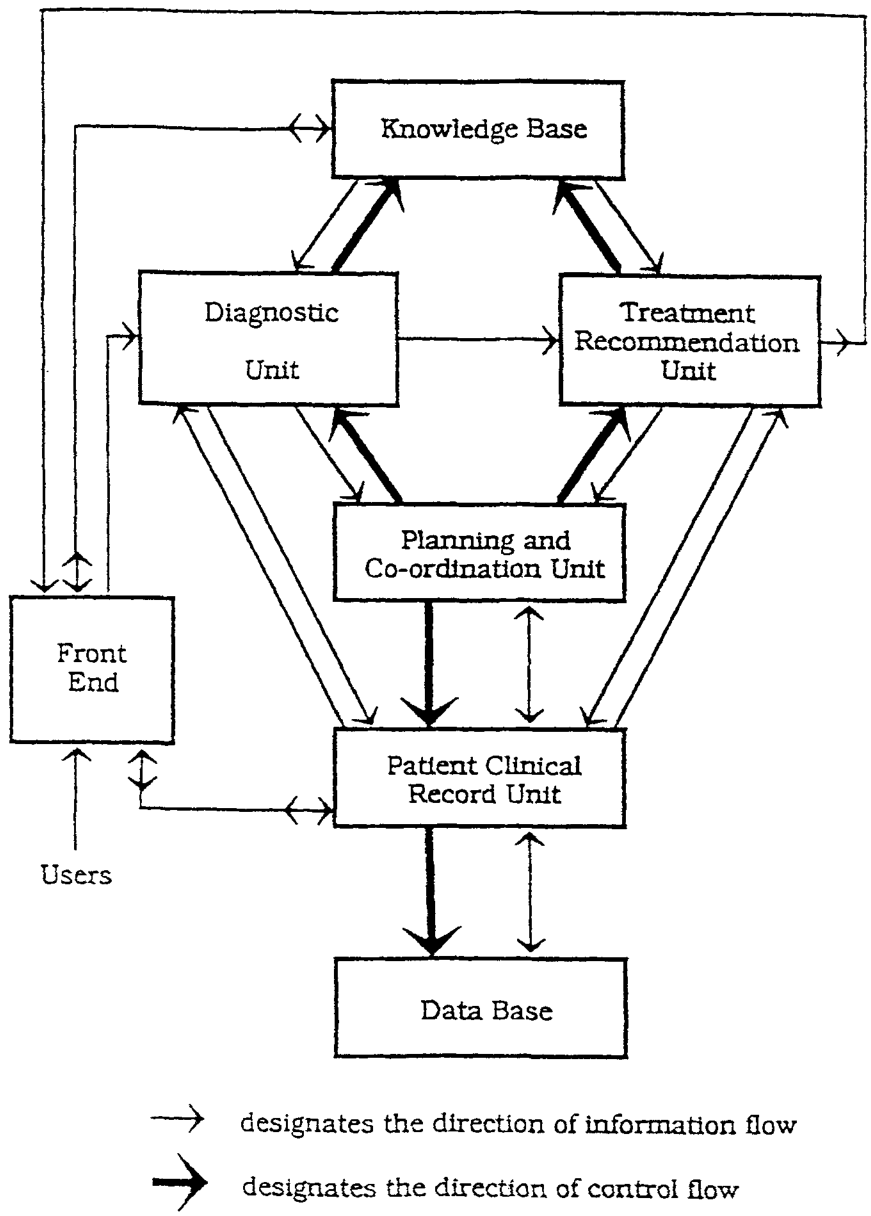

Figure 1. The basic configuration of CLINAID

and circumstances. Patients records are updated dynamically as the clinical process proceeds, and in some situations this also requires concurrency [35].

As CLINAID attempts to be a comprehensive medical consultation system, its knowledge base has to contain a large amount of medical expert knowledge. Taking the diagnostic unit as an example, CLINAID deals with a number of body systems, where each body system belongs to a different branch of medicine. The average size of the knowledge structure of a body system of a comprehensive medical system indicates that the problems of efficiency and correctness of inference are not insignificant:

Signs/symptoms per body system	200–400 approximately
Diseases per body system	100–400 approximately
The total number of body systems	11

Seven of the body systems listed in the CLINAID specification [47] formed part of the following list:

1. Cardiovascular
2. Respiratory
3. Central Nervous System
4. Muscular and Peripheral Nervous System
5. Renal
6. Endocrine
7. Blood and Reticulo-endothelial
8. Reproductive
9. Skeletal
10. Gastrointestinal
11. Psychological

A comparison with INTERNIST / CADUCEUS shows that in CLINAID we are faced with a formidable problem. INTERNIST deals only with internal medicine and contains several hundred diseases. In its size this corresponds to one body system of CLINAID. We are considering eleven body systems in the specification of our design. When the patient's signs and symptoms indicate candidate diseases from more than one system, the complexity of inference is formidable and parallelism is required for acceptable performance of the system. It will, however, be seen in the sequel that parallel inference can be performed also within one and each body system subunit of the diagnostic unit. Indeed, to have an acceptable speed of performance when a multiplicity of many-valued logic is used, some degree of real-time concurrency is inevitable.

3.2. The treatment recommendation unit

The purpose of the second unit is to advise about treatment, taking into the account the working diagnosis and the possible adverse effects [56] of each member of the family of prima facie treatments appropriate for each individual patient. This unit accepts the conclusion of the Diagnostic Unit as its input data. It also sends a request for the pharmacological drug history to the patient record unit. In order to make a treatment recommendation, it needs the experts' knowledge from the Knowledge Base and the past history of the patient provided by the Patient Clinical Record Unit.

3.3. Co-ordination and planning unit

The fourth unit provides the overall control of the clinical management activities as well as all the necessary auxiliary matching functions such as generating and unifying the communication protocols. In addition it has to contain the log of the co-ordinating process to facilitate monitoring and auditing of the operation of the whole system.

We have seen that each of these four units fulfils a certain global task. The global tasks co-operate concurrently and for this purpose the units are connected to each other by appropriately matched data and control flow interfaces which are structured within an additional unit called "whiteboard." It should be noted that each unit of Clinaid may consist of a number of centres and thus itself form a multi-centre fuzzy knowledge-based system.

4. Fuzzy relational architectures – a vehicle for achieving high parallelism in computing systems with interval based computations

4.1. Clinaid viewed as an instance of fuzzy relational architectures

There were two basic problems that precluded us from adapting conventional techniques used in the expert systems field, namely:

1. The large number of signs, symptoms and diseases used in Clinaid.

2. Each body system of Clinaid deals with a different medical field.

The first problem noted above is resolved by adequate parallel relational structures. The second one is more serious, as it is a problem that is both logical and conceptual: no amount of computational power can avoid problems caused by the *mixing of several distinct contexts*. Only a good, well structured design of a Knowledge-Based System based on logics with well defined semantics and epistemology can provide a remedy.

With regard to the currently used rule based techniques, the number of possible combinations of signs, symptoms, physical examinations, laboratory tests and diseases increases rapidly with each new body system. It is therefore not possible to represent such a large amount of information by means of production rules and reach a conclusion by searching a tree with these rules in the normal diagnostic time scale, using an on-line system.

For these reasons, we have radically departed from established practice and used fuzzy relational structures in most parts of CLINAID, not only for the processes of inference and knowledge representation, but have also included fuzzy relational dynamic protection structures in the data bases [25] and fuzzy relational methods of retrieval of data base objects [43, 50].

In order to deal with all the problems outlined above, CLINAID uses fuzzy relational inference based on the triangle relational products, which were introduced in 1977 by Bandler and Kohout [7, 15, 18]. CLINAID chooses to use fuzzy relations to capture this large amount of clinical knowledge in a structured way, and to make it easier to deal with parallelism that is essential for a multi-centre approach.

As suggested by the foregoing discussion, from the point of view of system architecture, our approach is grounded on two essential features that are built into the system [34, 38]:

1. Plurality of control and inferential centres.
2. Inference via triangular relational products.

The plurality of centres requires their *mutual co-operation*. Briefly, the plural centres co-operate as follows:

1. Each centre is adapted to dealing with a particular portion of the common task.
2. Each centre filters out different aspects of data, according to the context or to what its special competence is.

The power of this way of organising things comes jointly from the *specialisation* of the former and the *simplification* of the latter. How the *common task* is *subdivided* and how the centres are co-ordinated are of course delicate and crucial questions [28], but assuming that they can be achieved successfully then the advantages of subdivision are evident [38]:

1. Each centre deals only with a small portion of systemic knowledge.
2. The centres can operate in parallel.

These provide a major weapon against combinatorial explosion, with the following desirable consequences:

1. Inference and meta-inference are both expressible as relations.
2. Inference is computable in parallel.

This theme of parallel computation is one of the unifiers of the the plurality of centres and of relational inference. When dealing with multiple contexts or multiple environments [33], parallelism is essential, as are in this situation the needs for comparison and evaluation of several contexts or environments. We shall see that *mathematical relations* are an important theoretical as well as computational tool in parallel information processing. We use a plurality of mathematical relational systems, each system being based on a specific many-valued logic. All these systems form together a generic family that has in common, generic logical types of relational operations, such as **triangle** or **square relational products**. It should be understood that the essential condition placed on the properties of any fuzzy theory is that for non-fuzzy (crisp) data, all the theories must default into the same classical (2-valued) semantic theory of mathematical relations.

5. Fuzzy relational methods

5.1. Motivation for a mathematical theory

In this section we give an overview of the way in which certain unconventional types of relational products (introduced by Bandler and Kohout in mid 1970s) work, both from the mathematical side and in the concrete context of the medical domain. This is essential for a deeper understanding of the way the computational and knowledge representation structures work in CLINAID.

The mathematical definitions of three families of fuzzy products, namely of *triangle subproduct* $x(R \lhd S)z$, *triangle superproduct* $x(R \rhd S)z$ and *square product* $x(R \square S)z$ which are needed in the sequel, are presented in this section together with some examples of their use. A theoretical semantic device called the *checklist paradigm* which links the products with interval-based computations will be discussed in Section 6. Further detailed aspects of the theory and practice of **fuzzy relational products** and fast fuzzy relational algorithms [9, 19] that we use for the description, representation, and symbolic manipulation of knowledge can be found in two survey articles [44, 46] which also contain further bibliography. As a brief introduction to the relational architectures based on the triangle and square products, the reader might find [40] useful.

What is inadequately presented as 'relation theory' in the majority of books available, centres mostly upon *equivalence relations* and a couple of kinds (*ordinary* and *strict*) of *order relations*, all of which are characterized in terms of overlapping sets of special properties. Certain *other combinations* of the elementary properties have an importance which is much less generally understood [13, 14, 18, 19]. Once advantage is taken of these, new avenues for the **unified treatment** of various aspects of **parallel computation** [40] and *concurrent retrieval* [55] of information from multiple information sources can be provided [43]. This offers an additional advantage of introducing the **dynamic protection** of concurrent processes [31] in the same relational framework [39, 42, 59].

Since 1975 Bandler, Kohout and their coworkers have been developing the mathematical theory, computational algorithms and a systemic methodology of crisp and fuzzy relational products as well as being engaged in their practical utilization [7] in knowledge engineering, design of knowledge-based systems, information retrieval, analysis of medical and psychological data and in some business applications. The present section gives a brief exposition of definitions and results needed in the sequel, in the form which provides a much-needed unification of crisp and fuzzy relational theory (cf. Bandler and Kohout [18]). The pivotal point of the whole approach are the relational compositions of three distinct types, namely *triangle* sub- and super-product [4, 7, 8], and the *square* product. The triangle *subproduct* $R \lhd S$, the triangle *superproduct* $R \rhd S$, and *square product* $R \square S$ were first introduced by Bandler and Kohout in 1977, and their theory and applications have made substantial progress since then (cf. the survey in [44] which contains a comprehensive list of further references).

5.2. Triangle and square types of relational products

After introducing the characteristic features of the relational products of all four kinds, the section continues with a brief discussion of the logical meaning of different types of these products, together with some examples of their clinical meaning and use in the CLINAID architectures, again concerning both general and specific results.

A binary (2-place) relation R between two sets X and Y is a *predicate* with two empty slots. When an element x of X is put into the first slot and an element y of Y into the second, a grammatical statement ensues, which may be either true or false if the relation is crisp. For a fuzzy relation a degree of truth is assigned to that predicate. For any element x_i in X, the *afterset* x_iR is the subset of Y consisting of those elements to which x_i is related via R. Similarly, for any element y_j in Y, the *foreset* Ry_j is the subset of X consisting of those elements related by R to y_j. If R is a crisp relation, these are crisp sets; if R is fuzzy they are fuzzy sets, with membership degrees given by the degrees of the truth of the relation. The symbol R_{ij} will denote in the sequel the *degree* to which the ordered pair of elements (x_i, y_j) is related by the predicate R.

Where R is a relation from X to Y, and S a relation from Y to Z, a *product relation* $R*S$ is a relation from X to Z, determined by R and S. There are several types of product used to produce product-relations, each distinctive in its *intention* and *use*. But, when the relations are fuzzy, there is a further wide choice of realization for each of the four product kinds, because of the plethora of the many-valued logics based candidates for the role of *implication operator* and other connectives. It is hoped that our readers will appreciate the distinctions among the four main *logical types* of products (circle, both triangle types, and the square type), which follow in this section, and the way in which the requirement for an implication operator or an equivalence operator comes in. To see that each product type performs a **different logical action** on the intermediate sets is important, as *each logical type* of the product enforces a *distinct specific meaning* on the resulting product-relation $R*S$.

In order to explain clearly the need for, and the significance of, different logical types of relational products, we begin with crisp relations, and then extend these to fuzzy. First, we shall present the formal definition of each type of product, then a discussion with examples. The definition specifies each $R*S$, for various $*$, by saying exactly when this product-relation holds between an x and z.

Definition.

Circle product:	$x(R \circ S)z$	$\Leftrightarrow$	xR intersects Sz
Triangle Subproduct:	$x(R \triangleleft S)z$	$\Leftrightarrow$	$xR \subseteq Sz$
Triangle Superproduct:	$x(R \triangleright S)z$	$\Leftrightarrow$	$xR \supseteq Sz$
Square product:	$x(R \square S)z$	$\Leftrightarrow$	$xR = Sz$

It can be seen that the triangle and square products of the relations R and S composed over the 'middle set' Y have a *different logical standing* from the usual circle product $\circ$, as the former products involve the power set $\mathcal{P}(Y)$ in the composition of R and S, hence the Bandler-Kohout non-conventional relational compositions are constructs in second order logics.

It is clear that before these products can be of real service in knowledge representation or in any other kind of structural semantic modelling, they must be fuzzified. For their fuzzification, the many-valued logic based (fuzzy) *power set theories* are essential [4, 6]. Indeed, their adequate definition and further theoretical development were closely linked to the employment of new relational products [7]. Once the concept of the fuzzy powerset is clearly understood, the way of fuzzifying formulas defining relational compositions becomes obvious. Let us look at just one of these products to illustrate the idea. The fuzzy degree to which two elements x, z are related in $x(R \triangleleft S)z$ is given by the following formula:

$$\mu\Big\{(R \triangleleft S)(x,z)\Big\} = \mu_{R\triangleleft S}(x,z) = \mu_{xR\in\mathcal{P}(X)}(xR \triangleleft Sz) = \pi(xR \subseteq Sz)$$

This example reflects in its notation rather well the second order concepts that participate in forming the products of 'triangle' and 'square' logical type. Further knowledge of conceptual, quantificational and computational features of the foreset-afterset notation is, however, required for deeper appreciation of all the relevant logical issues [7, 18]. Fortunately, the parallel computational aspects of the relational products can be dealt with conveniently when the matrix relational notation is used. In this representation the parallelism of computational structures stands out more clearly than in the pure set-theoretical notation. The concrete involvement of various types of many valued logics also appears in a more transparent way in the matrix notation – and this is helpful in highlighting the interval character of all computations.

In order to switch over to the matrix notation, it may help to express the relational products in their pure logical form. In the logic formulas, R_{ij} will represent the fuzzy degree to which the statement $x_i R y_j$ is true.

$$\begin{aligned}
(R \circ S)_{ik} &= \bigvee\nolimits_j (R_{ij} \bigwedge S_{jk}) \\
(R \lhd S)_{ik} &= \bigwedge\nolimits_j (R_{ij} \rightarrow S_{jk}) \\
(R \rhd S)_{ik} &= \bigwedge\nolimits_j (R_{ij} \leftarrow S_{jk}) \\
(R \,\square\, S)_{ik} &= \bigwedge\nolimits_j (R_{ij} \equiv S_{jk})
\end{aligned}$$

The customary logical symbols for the logic connectives AND, OR, both *implications* and the *equivalence* in the above formulas represent the connectives of some many-valued logic, **chosen** according to the properties of the products required. Lack of space does not permit us to enter into details of choice from the repertory of various implication operators that are available for this purpose. The interested reader will find the definitions of the most useful implication operators employed in forming the triangle products in Bandler and Kohout [8, 15, 17].

It is important to distinguish what we call the *harsh* products (defined above) from a different family, the family of *mean* products. Given the general formula

$$(R \,@\, S)_{ik} ::= \#(R_{ij} * S_{jk})$$

by replacing the outer connective # by $\sum$ and normalizing the resulting product appropriately. The mean products are very important in some applications [5], although their mathematical theory does not take such a neat form as that of the harsh products. An example of use of the mean square product in CLINAID is presented in Section 8.3 below.

5.3. Applications

The triangle relational products together with fast fuzzy relational algorithms [9, 19] were applied to various practical problems in a number of scientific fields. For example in the analysis of medical diagnosis: diagnostic data and patient management processes [5, 8, 15, 56], medical sign and symptom comparison [23, 53]; information retrieval [26, 43, 55, 57], handwriting classification [51], and other areas [29, 40, 54].

In all these applications symbolic computations, where not only quantitative but also qualitative notions are involved, play a crucial role. The relational structures provide a computational interpretation and representation of the semiotic structures, which capture the denotational meaning of the concepts of a specific application domain, denoted objects being the concepts and processes that the computer is to emulate in a computational model.

The crucial issue for us now is, what is the diagnostic clinical interpretation of each of these new relational products and whether or not each distinct logical type (i.e. triangular sub or super-product, the square product) also has a **distinct clinical meaning**. That this is so is best demonstrated to a nonspecialist by a suitably chosen example. Let us look at an example of *diagnostic classification*. If R is the relation between *patients* and *individual symptoms*, and S a relation between *symptoms* and *diseases*, $R * S$ will quite naturally be a relation between *patients and diseases*. It is clear that, for this relational composition to make clinical sense, its structure should be conditioned by the intermediate set and the observatory and classificatory relations R and S, that entered into its computation, although these no longer appear in the result explicitly.

Where R and S have the clinical meaning as stated, the meanings of these product-relations are as follows (Bandler and Kohout, [15]):

$x(R \circ S)z$:	x has at least one symptom of illness z.
$x(R \triangleleft S)z$:	x's symptoms are among those which characterize z.
$x(R \triangleright S)z$:	x's symptoms include all those which characterize z.
$x(R \square S)z$:	x's symptoms are exactly those of illness z.

These product-relations faithfully reflect aspects of the thought sequence in a diagnostician's mind: $R \circ S$ arouses suspicion; $R \triangleleft S$ would deepen it but $R \triangleright S$ would clinch it, while $R \square S$ is perhaps reserved for the textbook cases.

5.4. Forming new relations by products

Product-relations formed by the relational products represent new entities composed from the original data. Their their specific semantics defines the conceptual meaning which is partially dependent on the conceptual meaning of the original data-relations.

Now, we can go further than that. If R is any relation (perhaps itself a product of other relations) from X to Y, we call R^T the *transposed* relation (often called converse, or sometime "inverse"), in which y is related by R^T to x exactly when x is related by R to y. This well known converse tells us nothing new in itself semantically, but participates in important constructions. In particular, $R * R^T$ is a relation from X to X, and $R^T * R$ a relation from Y to Y. Examples using the specific relations mentioned in Section 2.1 include:

$x_i(R \triangleleft R^T)x_k$:	patient x_i's symptoms are among these of x_k.
$x_i(R \square R^T)x_k$:	patient x_i has exactly the same symptoms as x_k.
$y_j(R^T \triangleleft R)y_l$:	whenever symptom y_j occurs, so does y_l (in this group of patients).
$y_j(S \square S^T)y_l$:	symptom y_j characterizes exactly the same diseases as does y_l.

Thus constructed relations might exhibit important relational properties revealing some important characteristics and interrelationships of the source of information from which these were generated. Hence, an important problem of scientific analysis of the computed new constructs is **whether**, and **which**, special *relational properties* these possess. Suitable methods for testing the presence of these are required.

5.5. A multiplicity of choice of many-valued relational structures and operations

The triangle and square products may be based on a large variety of many-valued logic implication operators; a practical question then arises, as to which many-valued set or relational theory is the best for a particular application and/or knowledge domain. Evaluation experiments performed in various applications [23, 52] conclusively show how essential it is to select such a fuzzy knowledge representation structure or inference/decision making method that would appropriately match the data/knowledge structures dictated by a particular application. The most important point that emerges from empirical studies is that the technique that should be employed in a specific application will crucially depend on the nature of data and knowledge involved. Inappropriate choice of the technique (be it probabilistic, Bayesian or based on a particular Fuzzy many-valued logic) may distort results beyond recognition. This applies equally to our choice of inference or decision method, as well as of knowledge elicitation or representation formal scheme.

6. Checklist paradigm

Interval based reasoning plays an increasingly important role in fuzzy and other many-valued extensions of crisp logic. To be of use in a diversity of application domains, the systems for interval-valued inference, however, require formal semantics that are not derived on an ad hoc basis but are firmly grounded in a sound logical epistemology. Such a formal semantics that is derived by means of an exact mathematical method, and which also has a sound ontological and epistemological base was provided by Bandler and Kohout by means of the so called **checklist paradigm** [8, 10, 11, 16]. The checklist paradigm has given interesting theoretical results, clarifying the role of fuzzy semantics in epistemological justification of genuine interval-based approximate reasoning methods in general, and also displaying under what conditions probabilistic and other point-based methods of inference might show to be inadequate.

The checklist paradigm generates *pairs of* distinct implication operators and other types of connectives that determine the end points of intervals thus providing formally and epistemologically justified systems of *interval-valued* approximate inference. Because of the context dependence of inference in the majority of medical problems which reflects in choice of logics, the checklist paradigm has turned out to be particularly relevant while choosing the logics and inferential procedures for CLINAID. Adequate semantic justification of the particular choice of many-valued logics was needed not only for inference but also for knowledge representation in interval-based relational structures of CLINAID. This also was provided by the checklist paradigm. This had significant impact on the choice of connectives for relational systems employed in parallel architectures of the kind discussed in Section 4 above. The checklist paradigm, together with fuzzy questionnaires and triangle products also played an important role in experimental identification and verification of knowledge structures that were used in CLINAID [33].

6.1. What is the checklist paradigm semantic model?

In its most general form, the checklist paradigm pairs the distinct connectives of the same logical type to provide the bounds for interval-valued approximate inference. The global structure imposed on the many-valued connectives by the certain type of checklist paradigm contracting measures is shown to be the $S_{2\times2\times2}$ group [45]. Checklist paradigm, however, is applicable not only to the components of the object language, such as logical operators and connectives, but also at the meta-level, thus providing an interval logic based semantics for various rules of inference. As shown in [16] Section 6, in addition to generating various multi-valued interval-based extensions of the semantics for modus ponens and modus tolens rules of inference, it also provides a justification and the proofs of validity of new interval-based rules (modes) of reasoning *denial* and *confirmation* (modus negans and modus confirmans) [12, 60]. These do not have a nontrivial analogy in Boolean crisp logic.

Let the *abstract checklist* [16] consist of n descriptive adjectives (descriptors/terms), expressing some potential features of the object to be evaluated, that are relevant to some particular class classification. The suitable formal classification criteria over the class properties provide the assertion of those terms on the checklist that apply to the object being assessed. The assessment of the object is then *summarised* by computing the total score of the object by dividing the "yes" answers by the total number of the questions (i.e. descriptor terms on the checklist). As this computation corresponds to the sigma count used in fuzzy logic, it is natural to interpret the proportion thus scored as a *fuzzy degree* of the agreement of the features of the object with the classification determined by the checklist, thus providing a fuzzy class over a domain of the class properties (by a class comprehension action).

Now, consider a single checklist used to give a degree of assent to two different objects A and B, where these abstract objects A, B are both some propositions [8]. For the general case, formally, let F be any logical propositional function of propositions A and B. Where i and j can each take the value 0 or 1, let $f(i,j)$ be the classical truth value of $F(A,B)$ that corresponds to the evaluation i for A and j for B; this also must be either 0 or 1. Let $u(i,j)$ be the ratio of the number in the ij-cell of the constraint table, to the grand total. Then what we have been agreeing is that *the fuzzy assessment of the truth of the proposition* $F(A,B)$ is

$$m(F) = \sum_{i,j} f(i,j)u(i,j).$$

As Bandler and Kohout have shown [8, 16] this gives a satisfactory way of assigning fuzzy values to compound propositions. But *truth-functionality* has been lost. It has been employed in the construction of the fuzzy assessments of the truth of a compound proposition but it disappears in the outcome. The same fuzzy values a and b can be assigned to the propositions A and B, without at all determining unique values for the compound propositions. The values of A and B tell us something but not everything about the values of their compounds. Just what they do tell us, is as we have said, the central question of the paradigm, to which we now return.

The four interior cells α_{00}, α_{01}, α_{10}, α_{11} of the constraint table constitute its *fine structure*; the margins r_0, r_1, c_0, c_1 constitute its *coarse structure* (see Figure 2).

As noted earlier, if the grand total is supposed to be given beforehand, there are three pieces of information in the fine structure, only two in the coarse. Loss of information is caused by the process of contraction/restriction of the information concerning the object by the assessment operator m. This assessment operator will also be called *contraction/approximation* measure in the sequel.

The fine structure gives us the appropriate fuzzy assessments for all propositional functions of A and B; the coarse structure gives us only the fuzzy assessments of A and B themselves. Our central question is, *to what extent can the fine structure be reconstructed from the coarse?* Bandler and Kohout have shown [8, 10, 11, 16]) that the *coarse* structure imposes bounds upon the fine

	No for B	Yes for B	Row total	
No for A	a_{00}	a_{01}	r_0	
Yes for A	a_{10}	a_{11}	r_1	Define $a = r_1/n$
Column total	c_0	c_1	n	
		Define $b = c_1/n$		

Figure 2. Checklist paradigm of the assignment of fuzzy values

structure, without determining it completely. Hence, *associated with the various logical connectives between propositions* **are** *their extreme values.*

Thus one obtains the pair of logical connectives of the same type that yield the boundary points of the interval of imprecision. The inequality restricting the possible values of $m(F)$, written in its general form as:

$$\mathit{contop} \geq m(F) \geq \mathit{conbot}$$

where *con* is the name of connective represented by $f(i, j)$, takes its specific form for each logical type of the connective. There are 16 such inequalities, as there are 16 **logical types** of *con* [10, 11, 16].

Let us look now at some typical results. Choosing for the logical type of the connective *con* the *implication* and making the assessment of the fuzzy value of the truth of a proposition by the formula [8]

$$m_1(F) = 1 - (\alpha_{10}/n) \tag{4.1}$$

we obtain:

$$\min(1, 1 - a + b) \geq m_1(PLY) \geq \max(1 - a, b).$$

We can see that for the *contop* the *plytop* was chosen, and the checklist paradigm produced the Lukasiewicz implication operator, and the other bound (plybot) is the Kleene-Dienes implication operator.

Choosing for *con* the connective type **AND** we get:

$$\min(a, b) \geq m_1(\mathrm{AND}) \geq \max(0, a + b - 1).$$

Choosing for *con* the connective type **OR** we get:

$$\min(a + b - 1, 1) \geq m_1(\mathrm{OR}) \geq \max(a, b).$$

6.2. Logical types of connectives that enter the checklist paradigm semantic model

We have seen [10, 16] that for a given contraction/approximation measure $m_1(F)$, there are 16 inequalities linking the TOP and BOT types of connectives, as there are 16 logical types of connectives. Out of these 16 possible two-argument options, 10 are genuinely dependent on both arguments. The latter include the following logical types: *conjunction* (a OR b), *disjunction* (a AND b), *implications* ($a \longrightarrow b, a \longleftarrow b$), *non-implications* ($a \Leftarrow b, a \Rightarrow b$), *equivalence* ($a \equiv b$), *non-equivalence* ($a \bigoplus b$). The minimal set of necessary conditions that the individual logical types of connectives have to satisfy are:

1. Duality of AND and OR.
2. A non-implication is obtained by negating an implication, and vice versa.

These conditions give general logical constraints on the systems of connectives. Taking a specific constraint measure, one obtains more specific results. For example, m_1 defined above will yield 16 pairs *conbot* $\leq$ *contop* specifying the end points of the interval for all the possible logical types of connective, some of which are listed below:

Logical Type	**Valuation**
$\neg(a \longrightarrow b)$	$\max(b-a) \leq \min(1-a, b)$
$\neg(a \longleftarrow b)$	$\max(a-b) \leq \min(a, 1-b)$
$a \equiv b$	$\max(a+b-1, 1-(a+b)) \leq \min(1-a+b, 1-b+a)$
a EXCLUSIVE OR b	$\max(a-b, b-a) \leq \min(a+b, 2-(a+b))$
$\neg(a \text{ OR } b)$	$\max(0, 1-a-b) \leq \min(1-a, 1-b)$
$\neg(a \text{ AND } b)$	$\max(1-a, 1-b) \leq \min(1, 2-a-b)$

For the exhaustive listing of all 16 connectives see, e.g., [10, 16].

We may wish to know exactly how wide the gap between the largest and smallest values of the various operators is, because this tells us how much information is lost by using the coarse structure instead of the fine.

If we define the *unnormalized fuzziness of x* (cf. Bandler and Kohout 1978 [4]) as $\varphi x = \min(x, 1-x)$ then for x in the range $[0,1]$, φx is in the range $[0, .5]$, with value 0 iff and only iff x is *crisp*, and value .5 iff x is .5. We have:

Gap Theorem — Bandler and Kohout (1986), [16]:

$$\begin{aligned} a \text{ ANDTOP } b - a \text{ ANDBOT } b &= \\ a \text{ ORTOP } b - a \text{ ORBOT } b &= \\ a \text{ PLYTOP } b - a \text{ PLYBOT } b &= \min(\varphi a, \varphi b). \end{aligned}$$

$$\begin{aligned} a \text{ IFFTOP } b - a \text{ IFFBOT } b &= \\ a \text{ EORTOP } b - a \text{ EORBOT } b &= 2\min(\varphi a, \varphi b). \end{aligned}$$

The above formulas are epistemologically important, as they explain the relationship of interval logics to the concept of fuzziness. They clearly show that the greater the degree of fuzziness a proposition has, the wider is the interval that appears after performing on that proposition a step of interval-based inference.

6.3. Other checklist paradigm models

Referring to Figure 2, let us define $u_{lk} = \alpha_{lk}/n$ In this section, we shall examine some inequalities with contraction measures other than m_1 that yield interesting results (cf. Bandler and Kohout [8]). If for *con* type we choose an implication again, but only the evaluation "by performance" (that is, we are only concerned with the cases in which the evaluation of A is 1), we use $m_2 = u_{11}/(u_{10} + u_{11})$ and obtain the inequality [8]

$$\min(1, b/a) \geq m_2(F) \geq \max\Big(0, (a+b-1)/a\Big) \tag{4.2}$$

where *plytop* is in this instance the well-known G43 implication of Goguen-Gaines (cf., e.g., [8]). Still another contracting measure which distinguishes the proportion of satisfactions "by performance", $u(1,1)$, and "by default", $u(0,0) + u(0,1)$. Thus $m_3 = u_{11} \bigvee (u_{00} + u_{01})$ yields [8]

$$\max\Big[\min(a,b), 1-a\Big] \geq m_3(F) \geq \max(a+b-1, 1-a). \tag{4.3}$$

Two variations on measure 3 have turned out to be of interest [8]. One is its lower contrapositivization $m_4 = [u_{11} \bigvee (u_{00} + u_{01})] \bigvee [u_{00} \bigvee (u_{01} + u_{11})]$ which gives the following inequality:

$$\min\Big[\max(a+b-1, 1-a), \max(b, 1-a-b)\Big] \leq m_4 \leq \min\Big[\max(1-a, b), \kappa a, \kappa b\Big] \tag{4.4}$$

where *crispness* κa is dual to *fuzziness* and is given by the formula $\kappa a = 1 - \varphi a$ (cf. Bandler and Kohout [4, 8]).

The other arises by taking for the "performance" part the less conservative m_2, giving $m_5 = m_2 \bigvee u_{00} + u_{11}$. This yields

$$\max\Big[\min(1, b/a), 1-a\Big] \geq m_5 \geq \max\Big[(a+b-1)/a, 1-a\Big]. \tag{4.5}$$

For the proofs of the results presented in this section and further explanation see [8], Sections 5 and 6. We have seen that in their 1980 paper [8] Bandler and Kohout derived a variety of interesting results for the implications bounds by choosing different functions for $m_i(F)$.

6.4. An example of use of interval logics in Clinaid

We give here an example of using relations in Clinaid for suggesting new signs or symptoms the physician should check for, when other signs or symptoms of a patient were already identified as present by the physician. Commonly, when doctors are in the process of eliciting symptoms and physical signs, they are always asking and looking for closely related symptoms and physical signs. Therefore, there should be a fuzzy relation to represent the doctor's activity at the symptoms and physical signs level. This can be done by taking a set of possible signs SN and a set of possible symptoms SM and relating these in the four possible ways, as can be seen from the definitions of relations R_1, R_2, R_3, R_4 below. In general, one should not assume that all these relations are necessarily symmetric. Definition of the relation 'symptoms to symptoms'.

$$R_6 \in \mathcal{F}(SM \to SM).$$

The fuzzy relation R_6 shows the degree to which the specific symptoms are related to other symptoms. As there are symptoms that are closely related to each other, then if a patient shows a particular symptom, there is a possibility of showing another symptom with a specific degree of certainty given by the relation. The role of this fuzzy relation is to suggest possible further symptoms that a patient might show if he or she already has a particular symptom. This fuzzy relation assists with identification of possible undisclosed symptoms.

Definition of the relation 'signs to signs'.

$$R_7 \in \mathcal{F}(SN \rightarrow SN).$$

By analogy, this relation shows the degree to which the specific signs are related to other physical signs. The role of this relation is to suggest possible further physical signs that a patient might show if he or she has a particular physical sign. This relation assists with identification of undisclosed physical signs.

Definition of the relations 'signs to symptoms' and 'symptoms to signs'.

$$R_8 \in \mathcal{F}(SN \rightarrow SM); \quad R_9 \in \mathcal{F}(SM \rightarrow SN).$$

The relation R_8, (R_9) shows the degree to which signs (symptoms) are related to other symptoms (signs). The role of these relations is to suggest possible further physical symptoms (signs) that a patient might show if he or she has a particular physical sign (symptom). This relation assists with identification of physical signs or symptoms.

Relations R_6, R_7, R_8, R_9 are useful if the patient did not mention, for any reason, some of the symptoms, or if the clinician missed some of the physical signs during physical examination of the patient. The suggested physical signs or symptoms would remind the clinician about the possible existence of other physical signs or symptoms.

We shall illustrate the idea by a concrete example. Let us take relation $R \in \mathcal{F}(A \rightarrow B)$ from the set A to the set B such that both sets are from the cardiac body system, $A \in SN$ and $B \in SM$. The 'meaning' (semiotic descriptors) are given by:

Set A	**given symptoms**	**Set B**	**suggested symptoms**
a_1	sweating	b_1	feels cold
a_2	palpitations	b_2	fever
a_3	pallor	b_3	cough
a_4	dyspnoea		

The fuzzy relation R is determined by the following aftersets:

$$\begin{aligned} a_1R &= \{b_1/.7, b_2/.8\}; \\ a_2R &= \{b_1/.6, b_2/.7; b_3/.2\}; \\ a_3R &= \{b_1/.5, b_2/.7; b_3/.2\}; \\ a_4R &= \{b_2/.3; b_3/.7\}. \end{aligned}$$

Over some variable time-period the patient may exhibit one or more of the the following symptoms which in themselves may also oscillate with time: $A' = \{a'_1/.9,\ a'_2/.8,\ a'_4/.7\}$. The query of a physician is "*Assuming that today, the patient is sweating and has palpitations occurring together, or may have cough, what other symptoms can be expected and to what degree?*" Formally the query can be expressed as: $(a'_1 \wedge a_2) \vee a_4 = ?$

Let us use the measure m_1 as the contracting measure characterising the nature of the loss of information in the components of the fuzzy statement. This will yield the interval logics as follows:

Lower base: $x \wedge y = \min(x, y)$; $x \vee y = \max(x, y)$.
Upper base: $x \wedge y = \max(0, x + y - 1)$; $x \vee y = \min(1, x + y)$.

It should be noted that the lower base is formed by the connectives of the *mindiag* and the upper base by the connectives of the *maxdiag* type [10, 16], yielding the bottom and the top value of the interval, respectively.

The result of computation of the query of our example in given below, where the expected value is computed by the connectives $x.y$ and $x + y - xy$.

Suggested symptoms	Interval	Expected value
Feels cold	$[0, .6]$	.3
Fever	$[.4, .7]$	.6
Cough	$[.6, .9]$	.6

7. The global description of the activity structure of Clinaid

In Section 3 we presented a general overview of the Clinaid architecture. This was supplemented by an overview of relational structures that have been used in specification, definition and representation of Clinaid. We have seen that concurrency and parallelism is the leading feature of Clinaid system. As this special issue is specifically concerned with the link of interval-based computing with parallel computing, we shall demonstrate how these two themes interplay in Clinaid by looking at the design of the *diagnostic unit* of Clinaid in greater detail. We shall see that in this unit both, parallelism and interval-based inference play a prominent role. In order to do so, we have to examine those specific features of context dependent diagnostic reasoning and concurrent decision making of a clinical expert that are emulated by Clinaid.

We have explained elsewhere [33], how the specification of activities of the Clinaid structures resulted from a careful scrutiny and formalization of clinical activities placed in the appropriate environment. In order to carry out this scrutiny systematically we have applied the Activity Structures methodology [30–32]. As this involves substantial knowledge elicitation and acquisition issues, we cannot discuss it here in its entirety (cf. [21, 24] for further details). By looking at the dynamics of the medical diagnostic process we just isolate those conceptual features that are essential for the problem and have to be faithfully transferred into the final product—Clinaid. These comprise *concurrent evaluation* of competing hypotheses, *context dependence* of inference, and *incompleteness* of approximate clinical data.

7.1. Dynamics of medical diagnostic process

The activity structure of Clinaid mirrors the clinician's activity [37, 41]. At least the following four components applied iteratively, characterise the clinicians' activity while treating a patient:

1. Observation.

2. Conceptual classification and filtration of relevant observational data for (3).

3. Decision.

All these basic components interact strongly with one another. For this reason the characteristics and the concrete contents of the knowledge representation structure will be strongly affected by every one of them. This interaction of the structures manifests itself in the character of the dynamic process of inference: not only static logic components but also their dynamic interactions generate important logical properties. One of these properties is *context dependence* of logical inference in a clinical context. Context dependence has direct impact not only on the correctness of inference, but necessarily brings about the **multiplicity** of contexts in which the inference process should operate correctly.

7.2. Diagnostic knowledge structures with a multiplicity of contexts

The majority of extant medical expert systems just deal with a limited context, the largest domain of knowledge being just a single medical field. The restrictions are not, however, only due to the size of the field, although this is commonly believed. The problem of conventional expert systems goes deeper than that, being in its essence *conceptual*: their knowledge bases and inference engines cannot mix easily the knowledge from several fields. HEARSAY was the first system that dealt with multiple contexts by introducing a multi-centre architecture.

The problems that have to be dealt with are logical as well as methodological. It was the framework of *Activity Structures* that provided the adequate tools to tackle these problems. Within the framework of Activity Structures [20, 31] knowledge-based systems of architecture of CLINAID has evolved into an architecture with the following features:

1. It is capable of dealing with a *multiplicity* of contexts.
2. It is capable of utilizing the deep knowledge concerning *individual contexts*, taking into account cross-contextual similarities and differences.

Let us briefly look at the features of clinical expertize that conditioned this development. Experience shows that the dynamic paths of inference in diagnosis will be different if performed by an inexperienced medical student (despite her/his textbook knowledge) from that performed by an experienced clinician. One may ask the question why this is so?

Superficially, it would seem that the relation between signs, symptoms and illnesses suffices for performing an adequate diagnostic inference. However, the investigation of the logical structure of the medical knowledge shows that this is not the case. This leads to the following problems [36, 48]:

1. *Problem of INCOMPLETENESS:*
   Not all signs and symptoms characterising the disease are always observable or present.
2. *Problem of LOCALITY OF INFERENCE:*
   Not all signs and symptoms are relevant in a particular given diagnostic context.
3. *Problem OF COMPLEXITY OF INFERENCE:*
   The complexity of exhaustive matching of all disease descriptors and indicators (e.g. signs, symptoms, tests, etc) may be forbidingly high.

Generally, if the multiplicity of contexts is ignored in the design of knowledge structures dealing with locality of inference, the resulting expert system may exhibit unpredictable 'lapses of correctness of inference'. Namely, in the wrong context, it will simply reach a wrong conclusion, which unfortunately cannot be easily detected. To deal with this problem adequately, it is necessary to introduce multiple contexts into the inference of knowledge-based systems. In CLINAID, we deal with this problem by partitioning the architecture into multiple-body systems (cf. Section 3 above) as well as using **syndromes** for diagnostic inference. In terms of functional activity structures [31], the inferential process focuses its activity into a particular area of locality, according to the context, which is *semantically* controlled by the *syndromes*. Once the inference is focused into one, or several body systems, then within each body system, the context is controlled by introducing an appropriate questioning strategy (Kohout, Gao, and Kalantar [49]) in which syndromes play an essential role.

Thus the system moves from ignorance to adequate knowledge. In doing so, the system passes dynamically through various intermediate states. In these dynamic states, it has only partial information hence strategies of inference generate partial conclusions. The dynamics inference is such that several partial conclusions can be kept under consideration until further diagnostic activity can distinguish between them. As more than one possibility is being considered at once, the system implements a form of concurrent inference.

8. Multi-context fuzzy relational inference of Clinaid

8.1. The dynamics of expert knowledge: modes and strategies of inference

In order to use fuzzy relational products adequately in the structures of CLINAID, one has to examine the application of static conceptual links as well as the dynamics of the inference processes that reflect the specificities of the medical domains that are represented in CLINAID. It has transpired through substantial knowledge elicitation effort [20, 22] that the inferential processes as well as the knowledge represented have some common systemic features. It can be said that any inferential process in CLINAID is characterised by its conceptual *semantics* and *dynamics* [30, 31]. The functional activity structure capturing diagnostic knowledge can be divided (particularised) into two distinct parts:

1. The Semantic Descriptor Structure captures the semantic definitions of all relevant diagnostic concepts.
2. The Dynamic Descriptor Structure captures the clinician's inferential strategy and logic.

In our outline of the activity structures of the diagnostic unit we have to pay attention to both, dynamic descriptor structures and semantic descriptor structures. The dynamic descriptor structures capture the strategies of clinical inference. Semantic descriptor structures on the other hand are mostly concerned with determining the meaning of the individual disease descriptors within the body systems, thus determining the medical content of the knowledge base providing the diagnostic definitions of the individual body systems.

In the next section, we shall briefly explain the types of static structures—the structures that contain the diagnostic semantic descriptors of the elements of the body systems (such as signs, symptoms, syndromes, diseases etc). These static structures define descriptively or operationally (e.g. syntactically) the meaning of the concepts involved in Activity Structure (IPM–AS) of the concurrent centres of the Diagnostic unit.

8.2. Semantic descriptors of diagnostic knowledge and their levels of organization

In this subsection we shall present the semantic descriptors for activity structures concerned with the activities of the diagnostic unit. We concentrate here mostly on the semantic descriptors as these present the link between the structures and contexts of individual body systems and their environments. This determines the partitioning of the architecture of the diagnostic unit of CLINAID.

There are hundreds of symptoms and physical signs which are related to different body systems. These symptoms and physical signs do not have the same importance in each body system, some play a more prominent role in one body system than in another. A few may distinctly identify a particular body system.

Our method of dealing with complexity reduction is to subdivide the activities and so decrease the size of the problem to be resolved at any particular stage. This hierarchical manner of subdividing the activities reveals the structure of processes that can be executed in parallel and allows the inference to proceed rapidly. CLINAID uses classificatory knowledge in order to diagnose diseases. As more information is obtained from the patient and presented by the doctor to the system, groups of unrelated diseases can be eliminated from the diagnostic path. This process continues until the final diagnostic conclusion is reached. This emulates the activity of an experienced clinician in moving towards a final diagnosis through a series of distinct diagnostic stages or levels. Making explicit these diagnostic activity levels helps with reducing complexity of the components of the global activity, and in revealing the correct explicit ordering of inferential processes, at the same time. By using fuzzy relations for capturing these levels, knowledge can be organised, classified and defined in such a way that reference is made to a family of limited contexts that can be processed in parallel.

8.2.1. Diagnostic levels

This list presents the basic classification of the hierarchy of diagnostic levels.

General Diagnostic Levels:

1. Symptom and sign level.
2. Body system level.
3. Syndrome level.
4. General disease level.
5. Specific disease level.
6. Aetiological disease level.

The relations are created, represented and completed according to this level-based classification. Only the items of information which are relevant for reaching the conclusion of a specific level are utilized when each level is activated. The relations employed within a diagnostic level are particular to that level. They are made redundant after the process has passed through that diagnostic level. In this way, a fraction of the entire knowledge is used at any one time, and therefore the complexity of such a structure and the processing time are sharply reduced. The relations have to be linked in such a way that a continuous process is created. That is, the system must be able to reach a final conclusion by obtaining fresh items of information at each stage and using the next relevant relation to get to a further stage. The fresh items of information obtained at each stage have to be compatible with the contextual domain of the next relation in order for fuzzy operations to be applied. These fresh items of information are obtained by manipulating the activated relations at each stage. There are means available for protecting dynamically against incorrect activation ([31], Chapters 5–8) that have also been expressed by means of fuzzy relations and relational products and can be also manipulated by joint parallel and interval-based computations [42, 58].

At each level of the diagnostic process, different fuzzy relations are used to perform the inference because the *purpose* of each fuzzy relation is different. Hence internal information within each relation may also differ. For example, the use of symptoms and physical signs at level two (body system level) and three (syndrome level) are not exactly the same. The symptoms and physical signs at level two represent symptoms and physical signs of the whole human body, thus forming a relation which can identify possible body systems. On the other hand, symptoms and physical signs of level three represent all the symptoms and physical signs a syndrome consists of, and are meant to identify the relevant syndrome(s).

8.2.2. Semiotic descriptors and fuzzy relational knowledge structures of individual diagnostic levels

Let us list now, the medical meaning (semiotic descriptors) of all the sets entering into the relations of individual levels.

B ... a set of body systems
G ... a set of general diseases
D ... a set of specific diseases
I ... a set of investigations
P ... a set of patients
S ... a set of symptoms and physical signs
Y ... a set of syndromes

The above listed conceptual types are related by fuzzy relations at a number of levels as listed Section 8.2.1 above. We shall examine now the meaning and purpose of the relations of individual diagnostic levels.

Definition of the relation 'symptoms and signs to body systems'

$$R_1 \in \mathcal{F}(S \rightarrow B).$$

The relation R_1 relates the set S of symptoms and physical signs to individual body systems B. The aim of this relation is to select the body systems relevant to presented patient information. Therefore, the symptoms and physical signs which form this relation are those which are suitable to identify relevant body systems efficiently.

Definition of the relation 'symptoms and signs to syndromes'

$$R_{2,i} \in \mathcal{F}(S \rightarrow Y).$$

This is a family of fuzzy relations as there is a different relation $R_{2,i}$ for each of the body systems. The aim of this fuzzy relation is to identify possible syndromes. By identifying the appropriate syndromes, the complexity of search for diseases can be reduced, since a syndrome is a cluster of relevant symptoms and physical signs. The semantic type of each relation of this level is the same (from signs and symptoms to syndromes), regardless of which body system contains that fuzzy relation. But the domain and the range of each $R_{2,i}$ are different from the domains and ranges of other relations of this family since each belongs to a different body system. Also the degrees of symptoms, physical signs or general diseases differ, as these do not have the same certainty in each body system.

Definition of the relation 'syndromes to general diseases'

$$R_{3,i} \in \mathcal{F}(Y \to G).$$

The fuzzy relation $R_{3,i}$ shows the degree of relevance of syndromes Y to different general diseases G. The aim of this fuzzy relation is to identify the general diseases of the patient. There is a separate relation for each body system.

Definition of the relation from 'general diseases G to investigations I'

$$R_{4,i} \in \mathcal{F}(G \to I).$$

There are several possible types of investigation I, tests or examinations that can be performed to explore each general disease. Each investigation may contribute to a certain degree to a known general disease. In fact, the fuzzy relation R_4 shows for each general disease, the possible number of investigations with their degree of relevance. The aim of this fuzzy relation is to identify the possible investigations that can be performed for a particular general disease which ultimately can contribute to the identification of specific disease(s).

Definition of the relation from 'investigations results J to specific diseases D'

$$R_{5,i} \in \mathcal{F}(J \to D).$$

The results of the investigations, tests or examinations can lead to the identification of specific diseases. These investigation results can contribute by identifying specific diseases with a varying degree of certainty. The relation R5 shows the results of investigations which yield specific diseases with different degrees of relevance (certainty). The aim of this relation, accepting the results of an investigation, is to be able to confirm, reject or change the priority of the specific diseases that might be in the current working diagnoses set.

8.3. The use of interval relational inference in Clinaid: an example of diagnosis of a cardio-vascular problem

Here we shall look at a sample result of working diagnoses computed from real clinical data. The purpose of this example is to show how the intervals help to evaluate the reliability/plausibility of results. All the results presented in this illustration are computed with *mean square* relational products (cf. Section 5.2 above for their definition and clinical meaning). Although a number of interval logics based on the alternatives discussed in Section 3 have been tested, we shall use the logic based on m_1, i.e. the pair (Klene-Dienes ply, Lukasiewicz ply) combined by min into symmetric matching operators as defined below. These equivalence operators are used for computing the mean square products of the examples.

The computation proceeds through the following sequence:

Signs and symptoms of a patient entered into the system are matched with the knowledge structures of the cardio-vascular body system, passing through the levels:

(body system) $\Rightarrow$ syndromes $\Rightarrow$ general diseases $\Rightarrow$ working diagnoses.

A physician identified the following signs and symptoms in a patient and diagnosed 'Non-Penetrating Crush Injury of Chest'.

Symptoms: *ANXIETY (.70), PAIN IN BACK (.80), PAIN IN CHEST (.70);*
Signs: *COLD LIMBS (.80), BRADYCARDIA (.70).*

This set of signs and symptoms was presented to CLINAID. The top five diseases of the working diagnosis list computed by CLINAID are listed in the table below. There is a clear single winner, *nonpenetrating crush injury of chest* which is the correct diagnosis. It should be noted that it is not the absolute value but the ordering of the list that is significant. The magnitude of the value may vary, and is the function of the proportion of the input signs and symptoms to the total number of signs and symptoms attached to the disease. So, the less input information is available, the lower the value on the "working diagnoses list" is.

Disease	$\Box_{KD}$	$\Box_L$	$\Box_{KD} - \Box_L$
NON PENETRATING CRUSH INJURY	.462	.600	.138
PENETRATING INJURY	.380	.450	.070
MYOCARDIAL INFARCTION	.343	.382	.039
INFECTIVE MYOCARDITIS	.331	.369	.038
CARDIAC TAMPONADE	.329	.359	.030

KD: Kleene-Dienes based operator $a \leftrightarrow b = \min[\max(1-a, b), \max(a, 1-b)]$
L: Lukasiewicz equivalence operator $a \equiv b = \min(1-a+b, 1-b+a)$

Looking at the above example of diagnosis, one may wonder whether the use of the interval-valued computations is not a unnecessary refinement. The next, atypical example will demonstrate that this is not so.

The signs and symptoms of a patient with *Malignant Hypertension* were rated by a physician as follows:

Symptoms: *ANXIETY (.60), HEADACHE (.70).*
Signs: *BLOOD PRESSURE HIGH (.90), EXUDATES AND HAEMORRHAGES IN EYE FUNDUS (.80), LEFT VENTRICULAR ENLARGEMENT (.80), PAPILLOEDEMA IN EYE FUNDUS (.90).*

Disease	$\Box_{KD}$	$\Box_L$	$\Box_{KD} - \Box_L$
MALIGNANT HYPERTENSION	.360	.405	.045
HYPERTENSION	.356	.433	.077
ESSENTIAL HYPERTENSION	.322	.356	.034
INFECTIVE MYOCARDITIS	.300	.314	.014
MITRAL VALVE DISEASE	.287	.292	.005
NON PENETRATING CRUSH INJURY	.282	.309	.027

KD: Kleene-Dienes based operator $a \leftrightarrow b = \min[\max(1-a,b), \max(a,1-b)]$
L: Lukasiewicz equivalence operator $a \equiv b = \min(1-a+b, 1-b+a)$

It is interesting to note that the interval of values indicating *Hypertension* contains the interval indicating *Malignant Hypertension*. It should be noted that the former is less specific and may contain the latter. It can, however, be also seen that another more specific candidate, namely, *Essential Hypertension* is not contained in the interval of the values for *Hypertension* and must therefore be excluded.

8.4. Activity structure of the diagnostic unit

It is not always the case that all the signs and symptoms are presented to CLINAID at once, hence the system must be able to operate in an interactive incremental fashion. This, of course, complicates the activities of each body system of the Diagnostic Unit considerably. Let us briefly look at the dynamics of its functioning. Here we deal with Dynamic descriptor structures and with emulating the clinicians' inferential strategies dynamically as noted in Section 8.1 above.

The dynamics of interaction of a body system of CLINAID with one of its environments is depicted in Figure 3. This figure displays the Activity Graph of the dynamics of a body system. Each body system has its own independent activity of this kind processing information specific to that particular body system. The graph depicted in Figure 3 consists of four hyper-processes (i.e. tasks) that mutually interact. The arcs labelled "!" represent the outcome of the activities of the hyper-processes. Those marked "?" represent a query (e.g. request for some information or question, etc).

Table 1 specifies the types and the semantic meaning of the inputs and outputs of the hyper-processes. It should be noted that the hyper-process HP−1 represents the activity of the clinician, whereas HP−2, HP−3 and HP−4 represent the hyper-processes that belong to the CLINAID IPM−AS. The aim of HP−2 is to identify the possible body-systems affected by the ailment of the diagnosed patient, whereas HP−3 and HP−4 identify the relevant syndromes and the working diagnosis, respectively. Figure 3 also displays the messages exchanged between the diagnostic unit and other units of CLINAID.

9. Conclusions

The limitation of extant medical expert systems is both conceptual and logical: their knowledge bases and inference engines cannot mix easily knowledge from several fields without some adverse effects. Our KBS CLINAID deals with this problem by introducing a multi-centre architecture in the Diagnostic Unit. This naturally leads to parallel processing. A theoretical device called the *checklist paradigm* provides semantics for fuzzy connectives and also gives epistemological justification for the interval-valued inference that is used to great advantage in CLINAID's parallel inferential system.

In order to deal with the large number of signs, symptoms, diseases and body systems, CLINAID uses fuzzy relational inference based on the triangle relational products. This makes it easier to deal with parallelism that is essential for a multi-centre approach. The relational structures provide a computational interpretation and representation of the semiotic structures, which capture the denotational meaning of the concepts of a specific application domain, denoted

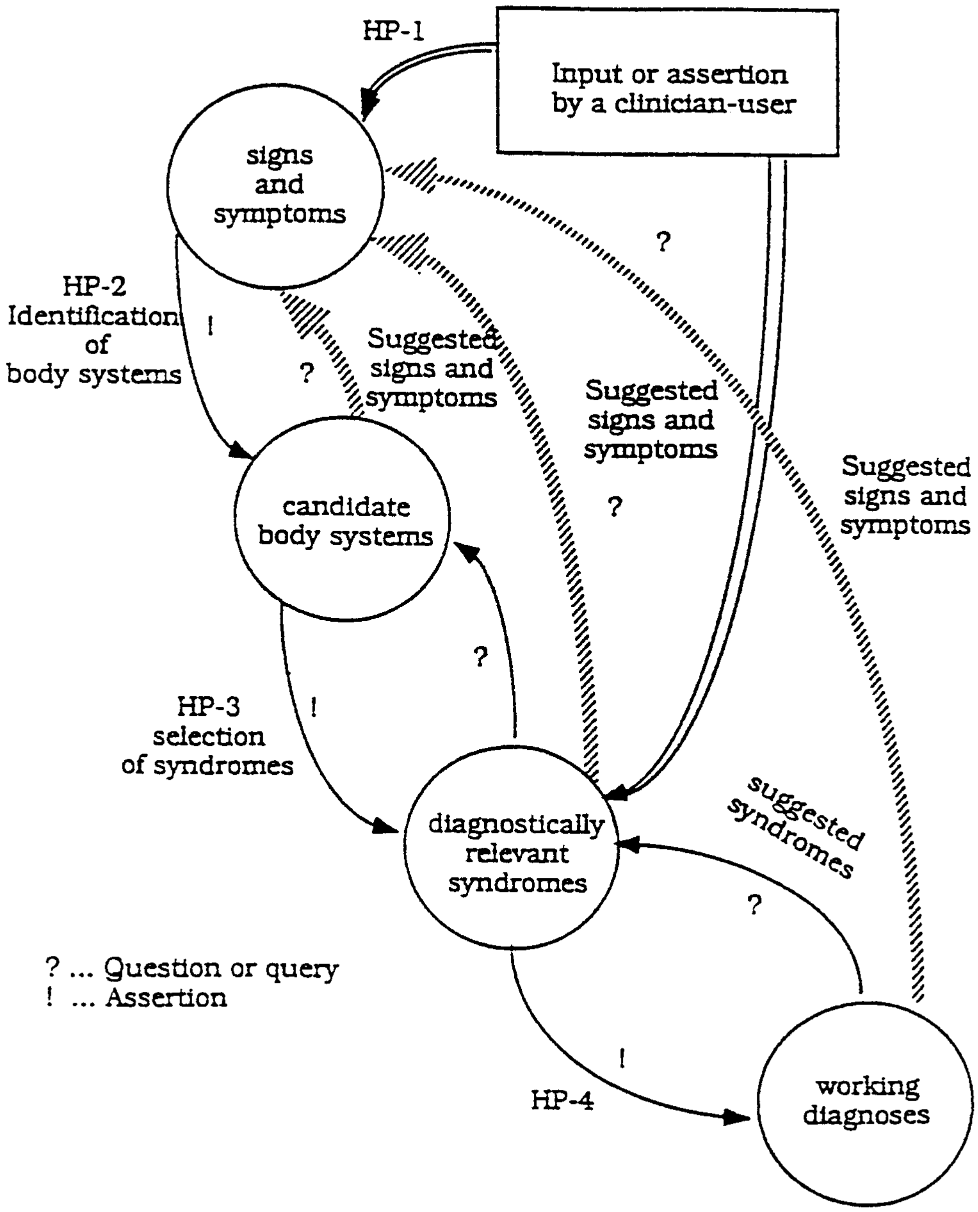

Figure 3. Activity graph of the dynamics of a body system of CLINAID
From: L.J. Kohout et al., An Overview of CLINAID, p. 188
In: Knowledge-Based Systems for Multiple Environments, Ashgate Press, 1992.

PURPOSE	Hyper-Process	INPUT	OUTPUT
Relevance filtering performed by the user of CLINAID	$HP-1$	Suggestions or questions presented by CLINAID to the user	data; user's assertions or rejections of the suggestions made by CLINAID
Identification of body systems	$HP-2$	Initial or asserted data from the clinician-user	candidate body systems
Selection of syndromes in a body system	HP_i-3	Initial or asserted data relevant to the selected body systems; filtered out from inputs and outputs of HP−1 and HP−2	Diagnostically relevant syndromes. Questions and suggestions to be presented to the user.
Forming the working diagnoses	HP_i-4	Diagnostically relevant syndromes; additional asserted signs and symptoms	A family of diseases "working diagnoses"

Table 1. Summary of Main Hyperprocesses of a Diagnostic Unit of CLINAID
Note that there is a separate HP_i-3 and HP_i-4 for each activated body system.

objects being the concepts and processes that the computer is to emulate in a computational model.

We have shown that concurrency and parallelism are the leading features of CLINAID system. By looking at the dynamics of the medical diagnostic process we have isolated those conceptual features that are essential for the problem and have to be faithfully transferred into the final product—CLINAID. These comprise *concurrent evaluation* of competing hypotheses, *context dependence* of inference, and *incompleteness* of approximate clinical data.

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