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CONTROL OF THE LINEAR DYNAMIC PLANT WITH INTERVALLY GIVEN PARAMETERS FROM THE GUARANTEE CONDITION OF THE REQUIRED ACCURACY OF THE SOLUTION

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The solution of the problem of the transformation of a linear dynamic plant with parameters known up to intervals into a given state is considered. The set of solutions every of which guarantees the required transformation accuracy is defined.

УПРАВЛЕНИЕ ЛИНЕЙНЫМ ДИНАМИЧЕСКИМ ОБЪЕКТОМ С ИНТЕРВАЛЬНО ЗАДАННЫМИ ПАРАМЕТРАМИ ИЗ УСЛОВИЯ ОБЕСПЕЧЕНИЯ ЗАДАННОЙ ТОЧНОСТИ РЕШЕНИЯ

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Рассматривается решение задачи перевода линейного динамического объекта с известными с точностью до интервалов параметрами в заданное состояние. Определяется множество решений, каждое из которых гарантирует заданную точность перевода.

The wide class of control plants can be represented by models of the form

$$\dot{\vec{\mathbf{x}}} = A\vec{\mathbf{x}}(t) + B\vec{\mathbf{u}}(t), \tag{1}$$

$$\vec{\mathbf{y}}(t) = C\vec{\mathbf{x}}(t), \tag{2}$$

where $\vec{\mathbf{x}} \in R^n$ is a vector of state variables; $\vec{\mathbf{u}} \in R^m$ is a vector of input variables; $\vec{\mathbf{y}} \in R^k$ is an output vector; $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{k\times n}$ are matrises which elements are directly connected with plant parameters.

Accordingly to the plant type, and the way of transforming to the description in the state space, matrices A, B, C may have distinct forms.

Then the problem of the control of a linear dynamic plant with one input and one output under interval uncertainty in values of plant parameters is considered.

Let in (1), (2)

$$A = \begin{pmatrix} a_{11} & 0 & \dots & 0 & \dots & 0 & 0 \\ 0 & a_{22} & \dots & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & a_{ii} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 & a_{nn} \end{pmatrix},$$

$$\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \vec{\mathbf{c}} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}.$$

where a_{ii} are roots of a characteristic polynomial; c_i are residues.

The given description is widely applyed in that case, when

$$a_{ii} \neq a_{jj}, \quad i \neq j,$$

that is, the system has simple roots.

It is required to find a programming control $u = u(t)$, transforming an plant from the given state $\vec{\mathbf{x}}(t_0) = \vec{\mathbf{x}}_0$ to the state $\vec{\mathbf{x}}(t_k) = \vec{\mathbf{x}}_k$, guaranteeing in this case the minimum of the criterion

$$I = \int_{t_0}^{t_k} u^2(t)dt.$$

In the case where description of the plant is known exactly an optimal control has the form [1]:

$$u(t) = \vec{\mathbf{b}}^{\mathsf{T}} \vec{\alpha}(t)/2,$$

where $\vec{\alpha}(t)$ is defined from the solution of the system of linear differential equations

$$\dot{\vec{\mathbf{z}}} = Q\vec{\mathbf{z}} \tag{3}$$

with boundary conditions

$$\begin{aligned} \vec{\mathbf{x}}(t_0) &= \vec{\mathbf{x}}_0, \\ \vec{\mathbf{x}}(t_k) &= \vec{\mathbf{x}}_k, \end{aligned} \tag{4}$$

where $\vec{\mathbf{z}}^{\mathsf{T}} = (x_1, \ldots, x_n, \alpha_1, \ldots, \alpha_n)$,

$$Q = \begin{pmatrix} A & \vec{\mathbf{b}} \cdot \vec{\mathbf{b}}^{\mathsf{T}}/2 \\ 0 & -A^{\mathsf{T}} \end{pmatrix}.$$

It is known that if the matrix A has eigenvalues λ_i, $i = 1, \ldots, n$, then the matrix Q has eigenvalues λ_i and $-\lambda_i$, $i = 1, \ldots, n$, to which eigenvectors $\vec{\mathbf{p}}_i \in R^{2n}$ and $\vec{\mathbf{q}}_i \in R^{2n}$ correspond. Then it can be shown that the optimal control has the form

$$u(t) = \left(\sum_{i=1}^{n} b_i \sum_{j=1}^{n} r_j p_{j,n+i} e^{\lambda_i t} + \omega_j q_{j,n+i} e^{-\lambda_i t} \right) /2,$$

where $p_{j,n+i}$, $q_{j,n+i}$ are $(n+i)$ th components of vectors $\vec{\mathbf{p}}_j$ and $\vec{\mathbf{q}}_j$ respectively; r_i, ω_i $(i = 1, \ldots, n)$ are parameters defined from equation (3).

If true values of plant parameters are known up to intervals, that is $a_{ij} \in [a_{ij}^-, a_{ij}^+]$, where a_{ij}^-, a_{ij}^+ are known lower and upper boundaries of the interval, then the matrix of parameters will be an interval matrix and

$$A \in A(I) \subset M^{n \times n}(IR),$$

$$A(I) = \begin{pmatrix} [a_{11}] & 0 & \ldots & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & [a_{ii}] & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 & [a_{nn}] \end{pmatrix}.$$

Then we shall say that a system with interval parameters has simple roots, if an intersection of intervals to which different parameters belong gives an empty set.

An interval analogue of the matrix Q has the form

$$Q(I) = \begin{pmatrix} A(I) & \vec{\mathbf{b}} \cdot \vec{\mathbf{b}}^{\mathsf{T}}/2 \\ 0 & -A^{\mathsf{T}}(I) \end{pmatrix},$$

and its eigenvalues are defined by the equation

$$[\lambda_i] = [a_{ii}], -[\lambda_i] = -[a_{ii}], \quad i = 1, \ldots, n,$$

and for every $a_{ii} \in [a_{ii}]$ and every $-a_i \in -[a_{ii}]$ there are respectively $\lambda_i \in [\lambda_i]$, $-\lambda_i \in -[\lambda_i]$ such that $\lambda_i = a_{ii}$, $-\lambda_i = -a_{ii}$.

The eiqenvectors $\vec{\mathbf{p}}_i(I)$, $\vec{\mathbf{q}}_i(I)$ corresponding to the eigenvalues $[\lambda_i]$, $-[\lambda_i]$ are found from the relations

$$\vec{\mathbf{p}}_i(I) = \left\{ \vec{\mathbf{p}}_i \in R^{2n} : (\exists \lambda_i \in [\lambda_i]) (\exists Q \in Q(I)) \left(Q\vec{\mathbf{p}}_i = \lambda_i \vec{\mathbf{p}}_i \right) \right\},$$

$$\vec{\mathbf{q}}_i(I) = \left\{ \vec{\mathbf{q}}_i \in R^{2n} : (\exists -\lambda_i \in -[\lambda_i]) (\exists Q \in Q(I)) \left(Q\vec{\mathbf{q}}_i = -\lambda_i \vec{\mathbf{q}}_i \right) \right\}.$$

In the presence of interval uncertainty with respect to plant parameters it is hardly reasonable to speak about solving problem in the sense in which it is understood for correctly known parameters, since solving this problem from the guarantee condition (4) is impossible because the set of control actions guaranteeing the validity of (4) at any $a_{ii} \in [a_{ii}]$ will be empty. Therefore, instead of (4) it is proposed to set the requirment of the validity of the equation

$$\vec{\mathbf{x}}(t_k) \in \vec{\mathbf{x}}_k(I) = [\vec{\mathbf{x}}_k^-, \vec{\mathbf{x}}_k^+],$$

where $\vec{\mathbf{x}}_k^-$, $\vec{\mathbf{x}}_k^+$ are given by an investigator from the sufficient accuracy condition of the solution of the problem.

It can be shown that the set of control actions satisfying the condition

$$\Omega_u = \left\{ u \in R^r : (\forall A \in A(I)) \left(\vec{\mathbf{x}}(t_0, t_k, A, U) \in \vec{\mathbf{x}}_k(I) \right) \right\},$$

will be of the form

$$\Omega_u = \left\{ u \in R' : u = \sum_{i=1}^{n} \omega_i e^{-\lambda_i t}/2,\ \forall \vec{\omega} \in \Omega_z,\ \lambda_i \in [a_{ii}] \right\},$$

where Ω_z is defined by the equation

$$\Omega_z = \left\{ \vec{\omega} \in R^n : (\forall G \in G(I))(G\vec{\omega} \in \vec{\mathbf{c}}(I)) \right\},$$

$$G(I) = (e^{[a_{ii}](t_k - t_0) - [a_{jj}]t_0} - e^{-[a_{ii}]t_k}) / ([a_{ii}] + [a_{jj}]),$$

$$\vec{\mathbf{c}}(I) = \vec{\mathbf{x}}_k(I) - \vec{\mathbf{x}}_0 e^{A(I)(t_k - t_0)}.$$

This allows to reduce resolving problem of finding admissible control actions guaranteeing desired accuracy on the finite state of the system to resolving system of linear algebraic equations with interval coefficients

$$G(I)\vec{\omega} = \vec{\mathbf{c}}(I)$$

and to defining the set Ω_z which is sometimes referred to as its "inner solution".

The set Ω_z represents a convex polygon with boundary hyperplanes defined by the system of inequalities [2]:

$$\Omega_z = \begin{cases} \max\limits_{G \in G(I)} G\vec{\omega} \le \vec{\mathbf{c}}^{+}, \\ \min\limits_{G \in G(I)} G\vec{\omega} \ge \vec{\mathbf{c}}^{-}, \end{cases}$$

where $\vec{\mathbf{c}}^{-}$, $\vec{\mathbf{c}}^{+}$ are boundary values of the vector $\vec{\mathbf{c}}(I)$.

The set Ω_z can be empty when the uncertainty of system parameters is so high that under the realization of the control on the plant, the transformation with a required accuracy cannot be performed. Taking this into account we have:

– the necessary condition of the existence of the set can be formulated in the following manner:

$$\vec{\mathbf{x}}_k^{+} - \vec{\mathbf{x}}_k^{-} \ge \max_{A \in A(I)} \left(e^{A(I)(t_k - t_0)} \cdot \vec{\mathbf{x}}_0 \right) - \min_{A \in A(I)} \left(e^{A(I)(t_k - t_0)} \cdot \vec{\mathbf{x}}_0 \right), \tag{5}$$

when the condition (5) is violated the interval and respectively the set Ω_z were found to be empty;

– the sufficient condition of the existence of the set consists in the validity of the equation

$$G\vec{\omega}_0 \subseteq \vec{\mathbf{c}}(I),$$

where the vector $\vec{\omega}_0$ is a solution of the system of equations

$$G_0\vec{\omega}_0 = \vec{\mathbf{c}}_0$$

$$\text{for } G_0 = \left\{\left(g_{ij}^{+} + g_{ij}^{-}\right)/2\right\}_{n\times n},$$

$$\vec{\mathbf{c}}_0^{\top} = \left(\left(c_1^{-} + c_1^{+}\right)/2, \ldots, \left(c_n^{+} + c_n^{-}\right)/2\right\}.$$

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