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INTERVAL APPROXIMATE REASONING FOR EXPERT SYSTEMS

Simeon J.Simoff

In the paper an interval system of continuous logic and the rules for constructing plausible inference based on it are considered.

ИНТЕРВАЛЬНЫЙ ПРАВДОПОДОБНЫЙ ЛОГИЧЕСКИЙ ВЫВОД ДЛЯ ЭКСПЕРТНЫХ СИСТЕМ

С.Симов

В работе предложена интервальная система непрерывной логики и правила построения правдоподобного вывода на ее основе.

1. Introduction

The recent experience in the sphere of knowledge engineering in artificial intelligence and, in particular, for expert systems (ES) light upon the necessity to extend the model of "omniscient" expert. A high-level professional can argue under incomplete, inexact and rapidly changing (even to opposite) information. Finally, expert's appropriate conclusions are not always strictly correct. Usually they are presumable, plural and reasonable with some degree of confidence that makes possible their further revision.

Classical logic deduction simulates strictly correct and generally valid clauses. Consequently, it is not suitable to express inexact and non-monotonic reasoning.

Inference in non-monotonic logics (Reiter's logic for default reasoning [6], McDermott-Doyle non-monotonic logics [3, 4], autoepistemic logics [5]) depends on the order of clauses. Though multivalued logics [2] offer additional "degrees of freedom" in inference, its interpretation and tracing are rather difficult.

Therefore, since non-monotonic system must discard its own conclusions, it must be able to modify the inference rules and to lock/unlock some of them.

Approximate reasoning in rule-based expert systems is based on different models of "inference confidence". This models make easier the sorting of conclusions over their degrees of confidence. The theoretical backgrounds of the variety of confidence estimators [1] are lacking that leads to their different heuristic interpretation.

In this paper is presented an interval continuous propositional logic for approximate reasoning.

2. Interval approximate reasoning

Let express an *inexact proposition* in the following form

$$p, [\alpha^-, \alpha^+], \tag{1}$$

where $p \in \mathbf{V}$, $\mathbf{V}$ is an unbounded countable set of clauses; $[\alpha^-, \alpha^+]$ is an interval estimator of logical truth (falsity), $\alpha^-, \alpha^+ \in S$, $\alpha^- \cdot \alpha^+ > 0$, $S = [S^-, S^+]$ is a symmetric scale of logical truth between "*False*" (**F** or S^-) and "*True*" (**T** or S^+), shown on Fig.1.

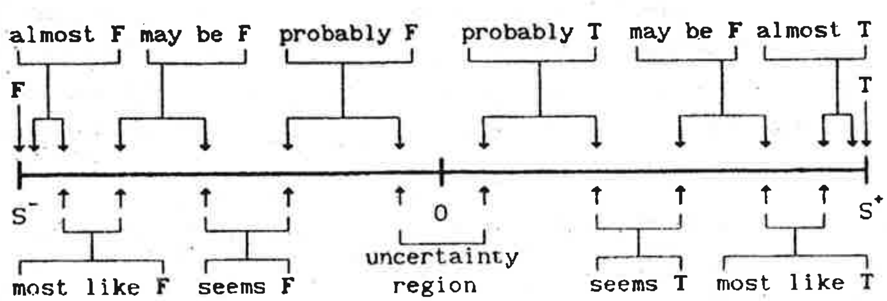

Fig.1. An example of verbal interpretation of scale S

Logical operations over propositions in form (1) are defined as follows.

Interval negation	
X	**¬X**
$p,\ [\alpha^-,\alpha^+]$	$\neg p,\ [-\alpha^+,-\alpha^-]$
$\neg p,\ [-\alpha^+,-\alpha^-]$	$p,\ [\alpha^-,\alpha^+]$

Interval conjunction		
X	**Y**	**X & Y**
$p,\ [\alpha^-,\alpha^+]$	$q,\ [\beta^-,\beta^+]$	$p\ \&\ q,\ [\min(\alpha^-,\beta^-),\min(\alpha^+,\beta^+)]$
$p,\ [\alpha^-,\alpha^+]$	$\neg q,\ [-\beta^+,-\beta^-]$	$p\ \&\ \neg q,\ [\min(\alpha^-,-\beta^+),\min(\alpha^+,-\beta^-)]$
$\neg p,\ [-\alpha^+,-\alpha^-]$	$q,\ [\beta^-,\beta^+]$	$\neg p\ \&\ q,\ [\min(-\alpha^+,\beta^-),\min(-\alpha^-,\beta^+)]$
$\neg p,\ [-\alpha^+,-\alpha^-]$	$\neg q,\ [-\beta^+,-\beta^-]$	$\neg p\ \&\ \neg q,\ [\min(-\alpha^+,-\beta^+),\min(-\alpha^-,-\beta^-)]$

Interval disjunction		
X	**Y**	**X ∨ Y**
$p,\ [\alpha^-,\alpha^+]$	$q,\ [\beta^-,\beta^+]$	$p \vee q,\ [\max(\alpha^-,\beta^-),\max(\alpha^+,\beta^+)]$
$p,\ [\alpha^-,\alpha^+]$	$\neg q,\ [-\beta^+,-\beta^-]$	$p \vee \neg q,\ [\max(\alpha^-,-\beta^+),\max(\alpha^+,-\beta^-)]$
$\neg p,\ [-\alpha^+,-\alpha^-]$	$q,\ [\beta^-,\beta^+]$	$\neg p \vee q,\ [\max(-\alpha^+,\beta^-),\max(-\alpha^-,\beta^+)]$
$\neg p,\ [-\alpha^+,-\alpha^-]$	$\neg q,\ [-\beta^+,-\beta^-]$	$\neg p \vee \neg q,\ [\max(-\alpha^+,-\beta^+),\max(-\alpha^-,-\beta^-)]$

where $[\alpha] = [\alpha^-,\alpha^+] \subset]0, S^+]$ and $[\beta] = [\beta^-,\beta^+] \subset]0, S^+]$

Complete logic equivalence of two inexact propositions $p, [\alpha^-,\alpha^+]$ and $q, [\beta^-,\beta^+]$ is defined as

$$((p,[\alpha^-,\alpha^+]) \approx (q,[\beta^-,\beta^+])) \equiv (p \approx q, [\gamma^-,\gamma^+]) \qquad (2)$$

where $\gamma^- = \alpha^- = \beta^-$, $\gamma^+ = \alpha^+ = \beta^+$. The central propositional connective $\equiv$ denotes "equality by definition". Complete logic equivalence in interval logic calculus is used in the context of identity.

Using the above presented operations and definition (2) are proved the following properties: *idempotency, commutativity, distributivity, associativity, complementarity, involution and D'Morgan rules,* for interval propositions.

If knowledge is represent in the form of inexact propositions (1) the **asserting deduction rule (modus ponens)** can be defined as

$$\frac{\begin{array}{l} p,\ [\alpha^-,\alpha^+] \Rightarrow q, [\beta^-,\beta^+] \\ p,\ [\gamma^-,\gamma^+] \end{array}}{q,\ [\chi^-,\chi^+]} \tag{3}$$

$$\text{where} \quad \chi^{-(+)} = \begin{cases} \frac{\beta^{-(+)}}{\alpha^{-(+)}} \cdot \gamma^{-(+)}, \ \text{if } \gamma^{-(+)} < \frac{\alpha^{-(+)}}{\beta^{-(+)}}, \ \text{else} \\ S^+ \end{cases} \tag{4}$$

for $[\alpha], [\beta]$ and $[\gamma] \subset]0, S^+]$.

In such formulation the extended modus ponens operates even in the case when intervals $[\gamma]$ and $[\alpha]$ does not match completely (and even does not intersect).

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