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OPTIMIZATION PROBLEMS FOR STATIC PLANTS UNDER UNCERTAINTY CONDITIONS

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This paper deals with optimization problems of static plants under interval uncertainty and its relationship with the solution of the linear interval system of equations.

ЗАДАЧИ ОПТИМИЗАЦИИ СТАТИЧЕСКИХ ОБЪЕКТОВ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ

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Рассматриваются задачи оптимизации статических объектов в условиях интервальной неопределенности и их связь с решением интервальной системы линейных уравнений.

When controlling economic systems and technological plants, the problem of finding an optimal solution is often arised. Evidently, the optimization of any technological process (TP) has as its ultimate goal finding the optimal solution, implemented as optimal control.

Let us assume that a multi-dimensional plant is described by the system of equations:

$$y^0 = A^0 x \tag{1}$$

where y^0 is a column vector of $(m \times 1)$ input parameters of the plant, x is a column vector of $(n \times 1)$ input actions, $A^0 \equiv [a^0_{ij}]$ is a matrix of $(m \times n)$ unknown parameters of the plant.

For plants with high uncertainty level, when is known only that unknown plant parameters α_{ij}^0 lie inside some given set: $A^0 \in \Omega_\alpha^0$, the dispersion of parameters α_{ij}^0, $i = 1, m$, $j = 1, n$, can be evaluated at the identification step by interval models

$$y = Ax, \tag{2}$$

where $A \in \Omega_\alpha \div \{\alpha_{ij}^- \leq \alpha_{ij} \leq \alpha_{ij}^+,\ i = 1, m,\ j = 1, n\}$ is a set of ordered intervals of model parameters.

When the models of form (2), defining the investigated system, are obtained and the scalar function $F(\overrightarrow{x})$ describing the optimality principle of the system is defined, then finding an optimum reduces to the solution of the non-interval conditional extremum problem

$$F(\overrightarrow{x}) \to \min_{\overrightarrow{x}} \tag{3}$$

under the constrains of the form

$$y = Ax, \quad A \in \Omega_\alpha, \quad y \in \Omega_y, \tag{4}$$

where $\Omega_y\ :\ y_T^- \leq y \leq y_T^+$ is an ordered set of intervals of technological constrains on the process.

In other words, the classical problem of the mathematical programming (3)–(4) can be reduced to the solution of the non-interval problem

$$F(\overrightarrow{x}) \to \min_{\overrightarrow{x} \in \Omega_x^*}, \tag{5}$$

where the set Ω_x^* is a set of "inner" solutions [1] of an interval linear system of the form

$$Ax = y_T. \tag{6}$$

The properties of the set Ω_x^*, such as convexity, connectivity, boundness, are described in papers [2], [3], [4]. Under the condition that $F(\overrightarrow{x})$ is a convex function, problem (5) is a convex programming problem which has an unique solution

$$\overrightarrow{x_0} = \arg \min_{\overrightarrow{x} \in \Omega_x^*} F(\overrightarrow{x}), \tag{7}$$

such that $F(\overrightarrow{x}) \geq F(\overrightarrow{x_0})$, for all $\overrightarrow{x} \in \Omega_x^*$ such that $\overrightarrow{x} \neq \overrightarrow{x_0}$. That is, in the optimization problem for the given objective function, the obtained set Ω_x^* is considered as the set of admissible solutions, i.e., solutions for which constraints of the form (4) are satisfied for every implementation $\overrightarrow{x} \in \Omega_x^*$ of an object [7].

Models (2) considered above can be thought to be a special case of more general model, where the plant is given by an intervally-valued function of vector argument $\overrightarrow{x}$:

$$y^0 = A^0 \varphi(\overrightarrow{x}), \tag{8}$$

where $\overrightarrow{x}$ is an input action vector, $\varphi(\overrightarrow{x})$ is a given collection of basic functions, A^0 are unknown parameters of the objective function. Therefore, the behaviour of an plant is expressable in the form

$$y_i(\overrightarrow{x}) \div \left\{ y_i : \min_{A \in \Omega_\alpha} \sum_{j=1}^{n} \alpha_{ij} \varphi_j(\overrightarrow{x}) \leq y_i \leq \max_{A \in \Omega_\alpha} \sum_{j=1}^{n} \alpha_{ij} \varphi_j(\overrightarrow{x}) \right\}, \tag{9}$$

where $i = \overline{1, m}$ and Ω_α is a convex set of model plant parameters.

In this case for the solution of an unconditional optimization problem with the quadratic criterion of the form

$$y(\overrightarrow{x}) = \alpha_0 + \overrightarrow{x}^T \overrightarrow{\alpha} + \overrightarrow{x}^T C \overrightarrow{x} \to \min_{\overrightarrow{x}}, \tag{10}$$

where α_0 is an interval, α is an interval vector, C is an interval matrix, we propose an approach based on necessary existence conditions of an extremum of the given function is proposed.

The solution is defined as values of variables Ω_x that guarantee the minimum to a given quadratic function (10), i.e., such that for all $\overrightarrow{x} \in \Omega_x$, there exists $C \in \Omega_C$, $\overrightarrow{\alpha} \in \Omega_\alpha$ for which the condition

$$2C\overrightarrow{x} + \overrightarrow{\alpha} = 0 \tag{11}$$

that is necessary for a minimum is satisfied.

That is, the optimization problem of a technological process with respect to an interval model of form (10) is formally reduced to the solution of the system of linear equations with interval coefficients and an interval right-hand side

$$-2C\overrightarrow{x} = \overrightarrow{\alpha}. \tag{12}$$

By a solution we mean the set of "external" solutions [3] taking into account the symmetry condition of the interval matrix C is the solution.

Basic features of the set of "external" solutions are described in papers [2], [3], [6] and others.

For sets of "external" solutions in the absence of the relationship between elements of different rows of the matrix of coefficients Ω'_x and in the presence of this relationship Ω_x the following inclusion is hold: $\Omega_x \subseteq \Omega'_x$.

In this case the equivalence condition of these sets can be written in the following form:

$$\Omega_x = \Omega'_x, \text{ if } \operatorname{sign} x_i = '+' \text{ or } \operatorname{sign} x_j = '-' \text{ for all } i, \quad i = \overline{1, n}. \tag{13}$$

The condition imposed on the sign of input values is not too severe. In many engineering problems, the parameters values x_i are greater than or equal to 0 because of some physical reasons, e.g., pressure, flow rate, temperature, material flow, etc., are always non-negative parameters. When this condition is valid there is a possibility to use previously developed algorithms for constructing sets of "external" solutions and thus solving an unconditional optimization problem with the interval quadratic criterion.

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