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# SYNTESIS OF TRANSPORT NETWORKS STRUCTURES UNDER CONDITIONS OF UNCERTAIN INITIAL INFORMATION

A.I.Demchenko, B.V.Peltsverger, O.V.Khavronin

Methods of synthesis of structures of transport networks under indefiniteness are considered. A classification of factors influencing the lack of uniocity of parameters of a transport network is proposed. To describe the idefiniteness available, the methods of interval analysis and fuzzy sets are used. An algorithm for solving the problem posed is given based on a decomposition approach.

# СИНТЕЗ СТУКТУР ТРАНСПОРТНЫХ СЕТЕЙ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ ИСХОДНОЙ ИНФОРМАЦИИ

А.И.Демченко, Б.В.Пельцвергер, О.В.Хавронин

Рассматриваются методы синтеза структур транспортных сетей в условии неопределенности. Предлагается классификация факторов, влияющих на неоднозначность параметров транспортной сети. Для описания имеющейся неопределенности используются методы интервального анализа и нечетких множеств. Предлагается алгоритм решения поставленной задачи, основанный на декомпозиционном подходе.

## Introduction

In this article the methods of transport networks structures synthesis are developed under the conditions of uncertain initial information.

Transport networks are characterized by the following uncertain factors: the power of source, time of network exploiting, information about geological conditions of building, directions of network development, modes of building etc.

The analysis of causes that give rise to uncertainty of parameters allows to discriminate two type of factors:

1) factors that are external for the model and act equally to its parameters. To these factors are related: time of network using, power of source, modes of building etc.
2) factors that are the consequence of incompleteness of engineering — geological information about building and exploitation conditions, the uncertainty that is arisen under the problem simplification. These factors influence the inside parameters of the model.

The first group of factors define the external uncertainty of the model, the second — the internal uncertainty of the model.

## 1. Problem formulation

Usually the initial information for the transport network synthesis problem is given in the form of graph $G = (V, E)$, where $V$ is the set of vertex, $E$ is the set of edged corresponding to admissible communications. A choice of transportation network structure is defined by the condition of minimization of creating and exploitation i.e. from problem

$$F(T) = \sum_{(i,j)\in T} (w(i,j) + v(i,j) \times y(i,j,T)) \to \min_{T}, \qquad (1.1)$$

where $w(i,j)$ is the communication $(i,j)$ creating cost, $v(i,j)$ is the unit product transportation cost along communication $(i,j)$ is the tree connecting source and sinks, $y(i,j,T)$ is the value of a flow along communication $(i,j)$ which determined uniquely for the given tree $T$, powers of source and sinks and the flow preservation condition. The problem (1.1) is $NP$-hard. In case of the external uncertainty the problem can be written down in the following form:

$$F(\hat{\alpha}, \hat{\beta}, T) = \sum_{(i,j)\in T} (\hat{\alpha} \cdot w(i,j) + \hat{\beta} \cdot v(i,j) \times y(i,j,T)), \quad T \in \Omega, \qquad (1.2)$$

where $\hat{\alpha}, \hat{\beta}$ are uncertain parameters, $\Omega$ is the set of spanning trees.

Under the interval uncertainty the problem is described as

$$F(T) = \sum_{(i,j)\in T} (\hat{w}(i,j) + \hat{v}(i,j) \times \hat{y}(i,j,T)), \quad T \in \Omega, \qquad (1.3)$$

where $\hat{w}(i,j), \hat{v}(i,j), \hat{y}(i,j,T)$ are uncertain parameters that are given on the edges of graph $G$.

For description of both the external and internal uncertainty in this paper are used the methods of interval analyses [1] and fuzzy sets [2]. The procedure of choice of optimal variants of network structure is based on the preference relations introduction and usage of the decomposition approach [3].

## 2. Algorithm of transport network synthesis under uncertainty

A problem (1.2) solving algorithm in the case of fuzzy evaluations is described in paper [4]. Because the interval evaluation is the particular case of fuzzy set described algorithm can be used for the problem (1.2) with interval evaluations solving.

For solving problem (1.3) with interval evaluations particular problems with polynomial complexity are introduced

$$u_1(T) = \sum_{(i,j)\in T} (\hat{w}(i,j)) \to \min, \quad T \in \Omega, \qquad (2.1)$$

$$u_2(T) = \sum_{(i,j)\in T} (\hat{v}(i,j) \times \hat{y}(i,j,T)) \to \min, \quad T \in \Omega, \qquad (2.2)$$

The problem solution can be obtained in the process of set efficient solution $P_u$ forming for the problem

$$(u_1(T), u_2(T)) \to \min, \quad T \in \Omega,$$

where $P_u$ is the set of effective solutions set in the space of criteria. Every solution $T$ of the problem (1.3) defines field

$$D(T) = [a_1, b_1] \times [a_2, b_2]$$



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4. Peltsverger, B.V. *hesis problem with* kibernetica, 4 (198
5. Gabow, H.N. *Tw* SIAM J. Comput..

in the criteria space, where $u_1(T) = [a_1, b_1]$, $u_2(T) = [a_2.b_2]$.

According to definition $T' \in P_u$ if there is no existed such $T''$ that $u_1(T'') \leq u_1(T')$ and $u_2(T'') \leq u_2(T')$ and besides one of inequalities is strict.

Solving the problem (1.3) consists of following steps:

1. $\{T'\}$ is constructed where $\{T'\}$ is a set shortest path tree that have non-dominating evaluations (interval problem (2.2) is solved).

$$b = \max_{T \in \{T'\}} \{b_1\}, \quad \text{where} \quad u_1(T) = [a_1, b_1].$$

2.The spanning trees $(T'')$ are generated in the order of $a_1$ increasing until $a_1 < b$ where $u_1(T) = [a_1, b_1]$, $T \in \{T''\}$. For the generation algorithm of Gabow is used [5].

The other spanning trees can't be effective solutions. The set of solution that are nonmodulated by the cost of construction and exploitation of transport network is formed during effective spanning trees generation. The problem (1.3) with fuzzy evaluations of alternatives can be reduced to the problem (1.3) with interval evaluations. On the basis of suggested approach the software is elaborated. This software allows to generate the set of nondominating uncertainty. It is used for networks designed for the oil field exploring in West Siberia.

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