Interval Computations
No 3(5), 1992

ALGORITHM FOR EXPERIMENTAL ZERO-ORDER OPTIMIZATION FOR PLANT WITH BOUNDED AMPLITUDE ERRORS

Alexander F.Bochkov and Lubov A.Yakovleva

This paper deals with the problem of the experimental optimization of plant with interval uncertainty in a criterion. Modifications of zero-order algorithm, allowed to supress "yaws" during the searching the extremum, are proposed.

АЛГОРИТМ ЭКСПЕРИМЕНТАЛЬНОЙ ОПТИМИЗАЦИИ НУЛЕВОГО ПОРЯДКА ДЛЯ ОБЪЕКТОВ С ПОМЕХАМИ ОГРАНИЧЕННОЙ АМПЛИТУДЫ

А.Ф.Бочков, Л.А.Яковлева

В статье рассматривается задача экспериментальной оптимизации объектов с интервальной неопределенностью в критерии. Предлагаются модификации алгоритмов нулевого порядка, позволяющие устранить "рыскания" при поиске экстремума.

When a series of practical problems being solved, the problem of a choice of optimal values of different parameters, that is a problem of solving optimization tasks, arises.

In the cases where the information about the considered process is not sufficient or the process is so complicated that it is impossible to obtain its analytical description, we made a recourse to experimental optimization methods among which there are well-known methods: Gauss–Seidel, simplex-method, random search and others [1]. However under the implementation of mentioned algorithms on real plant, the trajectory of the motion to the extremum as a result of action errors, noise becomes non-optimal, that is so-called "yaw" of the algorithm arises.

As a consequence of this, the problem of developing algorithms allowing to find correctly the extremum in noisiness conditions is actual [2], [3].

In general the work of zero-order algorithms can be represented in the following form.

(1) The trial experiment at points $x_1, x_2, \dots x_n$ is carried out and output values $y_1, y_2, \dots y_n$ are measured. Suppose that $x \in R^m$, y is a scalar.

(2) The best (the worst) point $x_i^-(x_i^+)$ is defined as the solution of the problem

$$x_i^- = \operatorname{argmin}\{y_1, \dots, y_n\}, \tag{1}$$

$$x_i^+ = \operatorname{argmax}\{y_1, \dots, y_n\}. \tag{2}$$

(3) The direction of the motion is defined and the operation step is performed.

Steps 1, 2, 3 repeat.

Suppose that for the noisy plant it is required to solve the problem

$$y(x) \xrightarrow{x} \min \tag{3}$$

using the implementation of experimental optimization algorithms. Classical methods does not take into account that the measured output value $y'(x_i)$ for plant will have the form

$$y'(x_i) = y(x_i) + e, \tag{4}$$

where $y(x)$ is a true output value; e is an error, applied additively to the output.

Assume that an error e can take any values at the closed interval $[-\Delta, +\Delta]$ and the value Δ is known to the investigator. The variability of an error can have in this case both random and undetermined character. Such plant referred to as plant with the bounded in amplitude error or plant with interval uncertainty. Note that the true output value $y(x_i)$ with guarantee liesin the interval

$$y(x_i) \in [y(x_i)] = [y'(x_i) - \Delta, y(x_i) + \Delta]. \tag{5}$$

With regard to the undeterminancy of the output, values of the experiment can be represented as $x_1 \to [y(x_1)]$, $x_2 \to [y(x_2)], \ldots, x_n \to [y(x_n)]$ and problems (1) and (2) are the optimization problems under interval uncertainty conditions. A set of unimprovable solutions is taken as the solution of such problems. For finding the set, relations of indifference and preference are used [4].

For the discrete collection of points $x_1, \ldots, x_n$ the set of unimprovable solutions for problem (1) is defined by the condition

$$\Omega^-_{1x} \div \{x \mid y^-_i \leq \min\{y^+_1, \ldots, y^+_n\}\}, \tag{6}$$

where y^-_i, y^+_i are respectively the lower and the upper boundaries of the interval $[y(x_i)]$.

For problem (2) we have

$$\Omega^+_{1x} \div \{x \mid y^+_i \geq \max\{y^-_1, \ldots, y^-_n\}\}. \tag{7}$$

If the set Ω^-_{1x} (or Ω^+_{1x}) contains a unique point, then in the point the extremum is surely achieved and therefore we can pass to the following step of the algorithm. Otherwise, it is impossible to distinguish the best (the worst) point for a given undeterminancy level and it is reasonably to provide correcting experiments at points which enter in the domain Ω^-_{1x} (or Ω^+_{1x}) and pretend to the extremum.

With regard to the first series of tests, we can write

$$\begin{aligned} x_1 &\to [y_1(x_1)],\ [y_2(x_1)], \\ x_2 &\to [y_1(x_2)],\ [y_2(x_2)], \\ &\cdots\cdots\cdots\cdots\cdots \\ x_k &\to [y_1(x_k)],\ [y_2(x_k)], \end{aligned} \tag{8}$$

where k is the number of points, entering in Ω^-_{1x} (or Ω^+_{1x}).

Taking into account that $y(x_i) \in [y_1(x_i)]$, $y(x_i) \in [y_2(x_i)]$ we can write

$$\begin{aligned} x_1 &\to [y_1(x_1)] \cap [y_2(x_1)] = [y''(x_1)], \\ &\dots\dots\dots\dots\dots \\ x_k &\to [y_1(x_k)] \cap [y_2(x_k)] = [y''(x_k)], \end{aligned} \qquad (9)$$

that is, under performing parallel tests, the width of uncertainty intervals decreases and therefore the set Ω_x^- (or Ω_x^+) contains a lesser number of points. For points $i = \overline{1,k}$ and appropriate intervals $[y''(x_i)]$, problems (1) and (2) are solved and Ω_{2x}^- (or Ω_{2x}^+) is constructed. The procedure repeats until the sets Ω_{jx}^- (or Ω_{jx}^+) contain a unique point x_i^- (or x_i^+).

Evidently, these points will completely coincide with extremum points of problems (1) or (2) and therefore the algorithm trajectory will completely coincide with the trajectory of the motion to the extremum of the appropriate optimization method in the absence of disturbances for any amplitude of the error Δ.

References

1. Adler, Yu. P., Markova, E. V. and Granovsky, Yu. V. *Experiment planning under searching optimal conditions.* Nauka, Moscow, 1990 (in Russian).
2. Bochkov, A. F. and Yakovleva, L. A. *Experimental optimization under uncertainty conditions.* Moscow Institute of radiotechn., electronics and automatics, Moscow, 1991, pp. 10–17 (in Russian).
3. Bochkov, A. F. and Yakovleva, L. A. *Methods of the experimental optimization of plants with an interval output.* In: "Proceedings of All-Union Conf. on Actual Problems of Appl. Math., Saratov, 1991", part 1, Saratov, 1991, pp. 76–79 (in Russian).
4. Voshchinin, A. P. and Sotirov, G. R. *Optimization under uncertainty conditions.* Moscow Power Institute, Moscow, Tekhnika, Sofia, 1990 (in Russian).