A Review of the Book

Dobronets B.S., Shaydurov V.V. Two-sided Numerical Methods "Nauka" (Siberian Department), Novosibirsk, 1990. – 208 p.

Рецензия на книгу

Добронец Б.С., Шайдуров В.В. Двусторонние численные методы — Новосибирск : Наука, 1990. — 208 с.

Numerical methods to obtain guaranteed results can be divided into a priori ones (initiated by Archemedes) and a posteriori ones. The latter do not require any validity when evaluating an approximate solution but should use validated computations with such solution to determine its relation to the exact one.

This book deals with the a posteriori methods based on defects estimation via interval analysis as the only up-to date "tool" which is adequate to such problems.

This methods go back to Cauchy's estimations for solving linear algebraic systems, Tchaplygin's fork" (1919) for solving of ODEs and to the L.V. Kantorovich's result (1948): if the equation is "smooth", the approximate solution is not "in critical case" and the defect is "sufficient small", then there exists the exact solution near the approximate one.

The book containes the following sections.

Foreword by the editor (p.5).

Preface (p.6-10). "The authors put the aim to construct relatively simple and sufficiently effective algorithms allowing to add (strict) analysis of precision of the results obtained to common numerical software".

The approximate (pointwise) solution of the given continuous problem is continued over the whole domain (by splines). The ratio of the defect estimate to that calibration function yields the deviation of the approximate solution from the exact one.

Introduction (p.11-16) contains the survey of papers on interval analysis and the general step-by-step scheme of further research.

Chapter 1 "Auxiliary information" (p.17-55): the continuation of the historical survey; elements of interval analysis; interval splines; comparison theorems for linear operator equations.

Chapter 2 "Algebraic problems" (p.56–89): various problem statements for systems of linear algebraic equations inner and outer; a posteriori methods; iteration methods.

Chapter 3 "Integral equations" (p.89–108): a posteriori and iteration methods for the Fredholm and Volterra equations.

Chapter 4 "Initial-value problems for the first order ordinary differential equations" (p.109–124): a posteriori methods.

Chapter 5 "Boundary value problems for ODEs" (p.125–154): the equations of the second order including those with small parameter at the highest derivative.

Chapter 6 "Partial differential equations" (p.155–195): appropriate problems for elliptic, parabolic equations are considered.

Some numerical examples are given and, under appropriate conditions, the estimations of the form for widths of interval solutions are proved.

The bibliography (p.196-204) contains 161 items.

This book as a whole is an essential step in developing interval analysis and expanding its applications.