INTERVAL COMPUTATIONS - SUBJECT OF RESEARCH AND USEFUL TOOL

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The state of things in the field of interval computations - a research direction, including interval mathematics, development of the corresponding computer tools and applications in science and technology, - is briefly described in this paper. The idea of interval approach and its comparative merits are explained as well as development of this approach into an original research branch. The information about publications, scientific meetings, bibliographic activities, geography of the interval computations prevalence in the world and in the USSR are supplied.

<u>1. Two approaches to numerical computation.</u> The classical mathematics has accumulated a fair amount of numerical methods over the period of its development. Those methods were formulated in terms of abstract mathematical objects, the most important of them was real number. But the attempts to apply such methods directly would always lead to one and the same problem: in practical computations one can use only finite representations (i.e. chains of limited length signs) and not every real number has the exact finite representation. It refers to initial data as well as to intermediate results of numerical computations. In the end, the situation, when it's also impossible to obtain an exact finite representation of the final result, becomes quite typical. Hence it arises the problem of correlating between the computed finite approximate result and the true abstract solution of the posed task.

In the present time there are two approaches to solving this problem. Due to the first, instead of abstract mathematical objects (such as real numbers, vectors and matrices) are used as the approximations objects of the same class, which have the exact finite representation. For example, they may be rational numbers from a certain finite set which also form vectors and matrices. At the same time the computation is accompanied by manual error estimating (*an error* is a deviation of the approximate value from the exact value) for every operation taking part in the computation. This approach was rather successfully applied in the pre-computer era, when a person, carrying out the computation, could permit oneself to regard error estimating even for the intermediate result as an independent creative task and to use for its solving the whole amount of means he had: from the passing to extended precision of intermediate results to the account of peculiar properties of numbers and functions, taking part in the computation.

Wide usage of electronic computers in computing practice immediately showed the scantiness of the described approach - mass computations by rather complicated algorithms make it impossible to regard estimating of every intermediate result as an independent task, so a unified technique of such estimating is indispensable. The urge to extend this approach onto the solving of numerical tasks with the help of computers has brought about the attempts to get the analytical dependence of the result's error from initial data and rounding errors. Despite certain achievements in this way (connected primarily with J.Wilkinson's name), one can say at present that in the whole those attempts have failed to achieve the planned goal. Among the reasons for this failure one can point out the following: computations by the formulas, connecting initial data errors with the result's errors are inevitably carried out with some errors themselves; those formulas should contain parameters (precision, radix, etc.) characterizing the instruction set of the computer which will be used for computations - so it's impossible to apply the formulas for any other computer; it's also impossible to take into account the individual accuracy of constituents in these formulas - hence a rough estimation of the final result. But the main reason lies in the difficulty of obtaining the realistic estimation of the rather sophisticated computation result's error - complexity of such a task can be compared to the complexity of the numerical method's development itself.

Besides, there's a serious drawback in the idea of substituting the exact value by one approximation itself. The thing is that, in general, correctness of the numerical relation testing is not guaranteed. For example, if in a program there's an operator

if x < 0 then A else B,

and x is computed approximately, then you can't be sure about the correctness of branching, because of, for instance, the value 0.013 may be a substitute for the exact value of x, equal to -0.02.

Drawing the conclusions, one can say that in many cases the first approach proved to be useful (especially in linear algebra) for the research of comparative accuracy and stability of numerical methods. But it couldn't guarantee correctness of computation by fork algorithms (let's take into consideration that mainly algorithms are fork!). But the main point is that this approach didn't provide a reliable answer for the simplest and most natural question: *how many correct digits are there in the obtained numerical result?* Consequently, the user of a programming system for numerical applications can't be positively sure about accuracy of computing such a value as, for instance, exp(0.0001).

Finally, more and more critically-minded researchers have started posing the question which became the name of the work [2]: *can we trust the results of our computing?* Awareness of this question's acuteness made a lot of them (as well as the author of [2]) resort to the second approach - the interval approach.

The essence of this approach is in substituting the unknown exact value not for the only element of the same class, as in the first approach, but for the finitely representable set which contains this unknown element. This approach has got its name due to the fact that an interval, usually represented by a pair of rational numbers-boundaries, is the simplest type of finitely representable set, localizing the simplest abstract object - a real number.

To make further discussion more convenient, let's make use of the following (non-traditional) terms: we'll refer to an abstract unknown mathematical object as to a *kernel*, the set containing it will be called a *shell*, and a pair (kernel, shell) will be called a *locus*. Thus locus is akin to an undamaged nut, the kernel of which can be operated with only by dealing with its shell.

Within the framework of interval approach initial data and intermediate results of computations are represented as locuses (with substituting the unknown values for the shells approximating them as precisely as possible); operations are performed only on the shells. At the same time all operations (first of all arithmetical) are defined in such a way that the result of the exact operation on kernels of arguments, which really does not computed, will inevitably belong to the computed shell-result. Computation aimed at obtaining a shell for the kernel solution of the posed mathematical task (at its localizing) is usually called *an interval computation* (although it would be more correct to call it *localizational*).

The interval approach allows to take into account all kinds of computing process's errors by a universal method: approximately known initial data are contained in shells which with a guarantee include the exact value, rounding errors only make the shells of intermediate results wider, and a halt of an infinite mathematical process limits the accuracy of result's localizing, but it doesn't cause the absence of the guarantee. The main merits of interval approach are: automatic control of all kinds of errors in the process of computation itself and the guaranteed accuracy of result (if, for example, you have obtained the result [1.71718, 1.71901] thus correctness of its first three digits is guaranteed). Now numerical relations can be tested by the correct way (it is enough to envisage in a program all cases of mutual location of compared values' shells). Peculiarities of a concrete computer are taken into consideration only while programming of operations themselves. Developer of a numerical program may ignore these peculiarities. Finally, individual accuracy of operands directly influences accuracy of every interval operation's result (the so-called property of inclusion monotonicity).

So, the interval approach is devoid of the major drawbacks of the first approach. One can also point out an important additional advantage: if one and the same task is solved by various numerical methods, then results of computations by these methods can be intersected in order to: 1) select a more accurate method, 2) to make the final result (intersection proper) more accurate than the one obtained by one of the methods taken separately, 3) to control the development and the programming of the methods (an empty intersection testifies about mistakes). One can see, that the second approach, undoubtedly, has more advantages than the first approach. But are there any drawbacks in it? Certainly, they exist certain amount of drawbacks but they are incomparable with advantages. The major drawback is unjustified dropping of accuracy when dependent variables or constants take part in the computation. This effect can be shown in a simplest example. According to the rules of interval arithmetic subtraction of intervals is defined in the following way:

$$[a, b] - [c, d] = [a-d, b-c].$$

Consequently, while performing the operation X-X on the interval X, the width of which is not equal to zero, the exact result [0, 0] will never be obtained. It will always be wider, although it will certainly contain [0, 0]. The reason of this phenomenon lies in the fact that operations on shells are always oriented for the "worst" case of the mutual position of kernels inside the operands - this is necessary to fulfill the reliability property. A lot of research works are devoted to partial and in some cases complete overcoming at this effect.

The other drawback of the second approach in comparison with the fist one is the greater complexity of basic operations and, as a consequence, greater expenditures of resources for the computation as a whole. One can completely get rid of this drawback by using special software and hardware (see paragraph 4).

<u>2. A short history of interval computations.</u> Despite the fact that the idea of abstract mathematical object's approximation

by including it in the finitely representable set has its roots in ancient times, its systematic development has begun only due to widespread adoption of electronic computers in computing practice. The first attempts of using numerical analysis in solving concrete tasks date back to the 50-60-s boundary. The founding monograph by R.E.Moore written in 1966 has become a decisive step in turning this idea into original mathematical branch. At first this branch was called *interval analysis*, then the term *interval mathematics* became more common due to the growing range of problems to solve.

According to the results of scientometrical research [4], procured on the basis of the quantitative analysis of the fullest bibliography (its earlier version was published in [5, 6]), the number of publications on interval computations was growing exponentially from the end of the 50-s till the beginning of the 80-s. Then it stabilized on the level approximately of 150 a year. The conclusion about as fast development as originality of the research branch was made in [4] (this is testified by a specific character of some scientometrical distributions).

In general, there are three directions in scientific and technological activity connected with interval computations:

- *mathematical direction* - including mathematical research of interval computations' problems,
- *computer direction* - considering issues of computer tools for performing interval computations development and usage,
- *applied direction* - connected with applications of interval mathematics' results and the corresponding computer tools in various fields of science and technology.

Let's very briefly consider each of these directions.

3. Interval mathematics. At present interval mathematics is first of all a numerical analysis based on the interval approach. Interval numerical analysis includes such fields as linear algebra, solutions of various equations and systems of equations, optimization etc., i.e. this repeats the traditional numerical analysis by names of parts. But there are some specific parts: for example, a lot of works are devoted to various methods of computing rational interval-valued functions. Some tasks of traditional numerical analysis have different interval formulations. Let's give a simplest example: the task of finding root x of the equation

$$a + x = b,$$

where a and b are real numbers. In interval formulation it may be set in two ways (A and B are intervals):

1) finding such interval X, that A + X = B,
2) finding such interval X, that X = B - A.

(Solution of the first task is the subset of the second task's solution).

As distinct from the traditional approach, set theoretic operations are widely used in interval algorithms: for example, an approximation obtained at (n-1)-th iteration, with the aim of accuracy raising, intersects with the approximation obtained at n-th iteration. Operands of these and arithmetic operations may be locuses formed of mathematical objects belonging to various classes: closed, open, half-open and infinite intervals on the real axis; circles, rectangles, sectors and other figures on the complex plane; balls, parallelepipeds and cones in multi-dimensional case. All the objects mentioned above may form vectors and matrices etc. Accuracy of a method is now characterized by some value corresponding to the "size" of the final result's shell. It can be estimated even before the computation itself by the way it is done at the first approach mentioned in paragraph 1.

Ones also refer to interval mathematics some researches not directly connected with the development of concrete numerical methods. For example, exposure of algebraic properties of a certain system of locuses or estimating the complexity of the solution of some interval task irrespective of the solving method (is, for instance, a task NP-hard?). There are also some works, in which concepts of interval mathematics and probabilistic-statistical ones are used together: for example, interval arithmetic with probabilistic and statistical distributions inside the intervals were studied.

Classical monographs in interval mathematics are [3, 7, 8], collections [9-11] are rather often quoted.

Last years inside the interval mathematics a tendency of combining in one method traditional and interval computations was outlined. Such a combining is done in the following way: for the beginning with the help of a traditional computation the approximate task solution is found, while as initial data are used the numbers which represent shells of exact initial data; then this solution by means of an interval computation is being spread over the whole initial data's domain of definition. Hence such methods get advantages of both approaches described in paragraph 1. These methods have got their name in western publications as *inclusion methods*. In the Soviet monograph [12], where they are a main research matter, these methods are called *two-sided*. It seems that proper interval in couple with combined methods would be named as *localizational*, because both of them differs from other methods by localizing (restricting) the exact solution of the corresponding classical mathematics' task. A branch of mathematics which develops and researches localizational methods would get the name *theory of localizational computations*.

<u>4. Computer tools for interval computations.</u> The first experiments with performing interval computations on electronic computers were made at the dawn of interval mathematics development. Implementing was usually done in the following way: interval operations were programmed in Algol-like or Fortran-like languages and mounted as a subroutine library. It was extremely inconvenient to use those subroutine libraries due to the lack of possibility to construct various data types, corresponding in such languages to classes of locuses, and to define infix operations on the elements of these types - most part of the program text was just a sequence of subroutine calls with swollen interfaces. Besides, such a method of implementation led to considerable (up to two orders) slowing

down of the program executing in comparison with common floating point computations.

Later the other approach was sampled: usage of languages with powerful means of types constructing and operations defining, such as Algol-68 and Ada. Terseness and expressiveness of program text had considerably increased; slowing down in this case didn't exceed one order. The main obstacles in widespread using of such languages were lack of their compilers at many modern mini- and microcomputers, and also - impossibility of effective implementing in these languages of operations with optimal direct rounding of results, that is necessary for most accurate account of rounding errors.

In the end of experimenting victory of the third approach has become. The essence of the approach is in implementing of numerical methods not in a universal programming language, but in a language specially designed for the numerical programming on the basis of interval approach. Such a language usually contains a sufficiently rich set of predefined types and operations, or has convenient and effective means of access to the predefined package, containing definitions of such types and operations and the necessary tools for constructing new types and operations. At first, a program written in such a language before execution was converted by a special preprocessor into some widespread programming language, then there appeared special translators. As the most important achievement here one may consider development of translators from languages *FORTRAN-SC* (for IBM-360/370) [13] and *PASCAL-SC* (for personal computers, for IBM PC in particular) [14]. Slowing down in comparison with common floating point computations didn't exceed several times in this case. Further growth of speed requires microprogram and hardware support of basic operations.

So a number of experimental developments are oriented on microprogram and hardware support. For example, there is an implementation of FORTRAN-SC, using the microprogram supported package ACRITH [13]; there are specially designed processors too. Taking into account the requirements of interval computations there were developed well known standards for computer arithmetic *ANSI/IEEE-754* and *ANSI/IEEE-854*. Today the vast majority of microprocessors having floating point instructions, are manufactured according to these standards.

At the same time continues development of software packages implementing various generalizations of interval arithmetic, computations with variable precision and with non-standard numeric representations, including elementary functions as well as libraries of numerical methods. These packages are programmed in various languages - from C to Prolog.

The progressive tendency of uniting numeric-interval and symbolic-analytical (computer-algebraic) programming systems is gaining power last time. Such uniting allows to combine merits of both approaches: to perform analytical evaluations, accompanying them by obtaining numeric results with guaranteed and controlled accuracy. The usage of symbolic-analytical evaluations enables to improve sometimes an accuracy of the interval computation itself.

One can learn about the state of things in the computer direction as a whole in 1987 by the survey [15].

<u>5. Applications of interval computations.</u> Interval computations have been a powerful and reliable tool in the various fields of science and technology over a relatively short period of time. First of all here one should mention mathematics itself. With the help of interval computations one was able to prove a number of important theorems requiring comparison of bulky numerical expressions in the process of proving. Since interval computations possess the proving power due to the property of guaranteeness, they were used for automation of the proving process (see the survey [16], and the paper [17]).

As for non-mathematical applications they have proved to be possible in any cases, where pointing out reasonable boundaries of some indefinite value became an adequate way of indeterminateness's formalization, contained in initial data or in a result. "Initial data lie in some boundaries (= belong to some sets) and are related by some formulas with the result.

Can one point out reasonable boundaries in which the results lies (= point out set, to which it belongs) the result?" - such a method of mathematical formalization turned out in many cases to be much simpler and more natural, than the one based only on probabilistic-statistical approach or on fuzzy sets theory.

Success of interval approach has brought about the situation when it is almost impossible to catch the spreading of interval computations in applied fields. For instance, the author of this paper is acquainted with some publications devoted to the applications of interval computations in metrology and machinery studies, chemistry and biology, radio electronics and electrical engineering, geodesy and geophysics, economic planning and prognostication, automatic control theory (maybe the largest amount of applications) and space research, computer aided design and computer graphics, development of general purpose software and even with organizing of fire-fighting and precious stones cutting. This list is certainly incomplete.

Obviously the rate of further interval computations' spreading in various fields of science and technology depends on accessibility of special and popular literature on this subject for "non-interval" specialists and of the corresponding well-developed software and hardware.

One can learn about some applications of interval computations by the survey [18].

6. Scientific meetings. Development of interval approach and its taking shape as an original research direction have promoted the need for holding scientific conferences specially devoted to its particular issues. The first international meeting on this subject - *Symposium on Interval Analysis* - was held in 1968 in Great Britain. Later, the most important meetings of similar kind were symposiums with the identical name *International Symposium on Interval Mathematics* held in West Germany in 1975 [9], 1980 [10], and 1985 [11]. The IBM company which had previously been doing interval research work held an international symposium in 1982. It took place in the USA and was called *A New Approach to Scientific Computation.*

After 1985 almost every year some important international conferences to a certain extent or wholly devoted to interval computations were held. Let us point out the most recent ones: *Reliability in Computing: International Workshop on the Role of Interval Methods in Scientific Computing* (USA, 1987), *International Symposium on Computer Arithmetic and Self-Validating Numerical Methods* (Switzerland, 1989), *International Symposium on Computer Arithmetic, Scientific Computation and Mathematical Modelling* (Bulgaria, 1990). Proceedings of all meetings mentioned above have been published.

Let us add that some countries like Poland and the USSR hold national scientific meetings on this subject. Besides, results of researches and applications interval computations are regularly reported about at the conferences of adjacent fields such as computer algebra, automatic control etc.

7. Publications. There are at least fifty books (monographs, collections, manuals on computer tools) of works) published in the whole world at the present moment, specially devoted to interval computations. Some of them, for example [7], can be used as a textbook in studying this direction within the framework of a special university course.

Periodicals are certainly of greatest interest for monitoring the current situation. The list of 10 journals having published the largest number of papers on this subject is given in [4]. Here is that list (in the order of the papers' amount decreasing): *"Computing", "Z. Angew. Math. Mech.", "Numer. Math.", "SIAM J. Numer. Anal.", "Wiss. Z. Techn. Hochsch. Leuna-Merseburg", "BIT", "Comm. ACM", "J. Math. Anal. Appl.", "ACM Trans. Math. Software", "IEEE Trans. Comput."* One can see, that the list is headed by the journals on numerical mathematics, the computer direction is also represented. As for publications on applications of interval computations, they are almost all published in editions which refer to the corresponding fields of applications.

The only periodical in the world specially devoted to interval computations – *Freiburger Intervall-Berichte* – was publishing in Freiburg (FRG) till 1988. Unfortunately, it has ceased its existence after 100 issues (10 issues a year) One can say that the present periodical - *Interval Computations* - has taken up the baton from Freiburger Intervall-Berichte.

Taking into account the trends revealed in [4] and the author's estimation of completeness of the investigated bibliography, the general number of published works significantly exceeds 3000.

8. Geography. Since the time of their springing up interval computations have made a long way not only as a research direction, but in the geographic sense too. It is hardly possible to name a country participating in big science - from Brazil to China - which does no research in the field of interval computations. Certainly, intensity of research varies rather drastically in different countries. The absolute leader is FRG, then goes the USA, the USSR, countries of Western and Eastern Europe, and so on.

The distribution of this field's leaders - the most active and quoted authors - on countries is also significant in this respect. According to [4], of eight revealed leaders five (*Alefeld G., Nickel K., Krawczyk R., Ratschek H., Kulisch U.*) work in FRG, three (*Moore R.E., Rall L.B., Hansen E.*) - in the USA. Soviet publications are hardly ever quoted by Western authors due to the language barrier and difficulties in access to them.

Institute for Applied Mathematics at the University of Karlsruhe (FRG) is the biggest center of systematic researches in the field of interval computations in the world, paying special attention to the development of the computer direction.

The total amount of researchers, who have published at least one work in this field is not less then 1000 in the whole world.

9. Bibliographic activities. The bibliography mentioned above is being maintained (on the computer medium as well). At present this work is done by Prof.Dr.H.Ratschek (Mathematisches Institut der Universität Düsseldorf, Universitätsstr. 1, 4000

Düsseldorf 1, FRG). A considerable amount of works pointed out in bibliography is contained in original or as copies in the same place - in *Interval Library*, open for all.

The Soviet center of the bibliographic activity is in the Computing Center of Siberian Branch of the USSR Academy of Sciences (USSR, 660036, Krasnoyarsk, Akademgorodok, Computing Center of Siberian Branch of the USSR Academy of Sciences, write to the name of Dr.B.S.Dobronetz). The bibliography has amounted to about 400 references, there are complete texts of the majority of works.

Finally, a specialized bibliography on the computer direction in interval computations is being compiled by the author of this paper. The bibliography includes about 350 references.

Both the bibliographies just mentioned above are maintained with the help of IBM PC and, correspondingly, are kept on computer medium.

10. Interval computations in the USSR. As it has already been pointed out the works of Soviet researchers are almost unknown abroad due to the language barrier and difficulties in obtaining them. Let us very briefly try to describe the state of things in the USSR because one of the tasks of this edition is establishing of relations relations in the field of interval computations.

As an original direction interval computations in the USSR have started developing since the middle of the 70-s. At present researches of mathematical problems as well as in the field of computer tools and applications are being carried out.

In the mathematical direction the research is under way in practically all parts of interval mathematics. But maybe the greatest number of works are devoted to the research of ordinary differential equations and the tasks of linear algebra. The so-called aposteriory interval analysis has been developed. It is a universal method allowing to effectively overcome the first drawback of interval approach mentioned in paragraph 1. There are original monographs on interval mathematics in Russian [12, 19], the monograph [8] was translated into Russian. There are special courses for students on the subject at some universities.

Computer direction is being rapidly extended: several packages of interval operations for various types of computers have been developed, an experimental translator from the language allowing to perform aposteriory-interval computations has been implemented, there is a number of theoretical works concerning to the language support and the computing process organization while executing interval computations on a computer (including parallelism issues), a certain experience in combined usage of interval and computer algebraic tools has been gained.

The sphere of applied researches has been getting wider. There have been some attempts to use interval computations in a considerable part of fields mentioned in paragraph 5. Obviously, electrical engineering, radio electronics and theory of automatic control are in the lead. A number of serious results have been obtained in the mathematics itself (based on usage of interval computations as proving computations [16, 17]).

The All-Union meetings on interval mathematics, at which the computer and applied directions are really represented too, are held annually. The first conference took place in Saratov in 1989. For the present materials of that conference have been published as the collection of theses so far. Unfortunately, the above mentioned conferences were not attended yet by foreign specialists. It is worth to note that Soviet specialists started participating in conferences on the subject held abroad only in 1990.

Geography of researches in the Soviet Union has been extending. At present interval computations are studied in Siberia, in the Urals, in Central Russia, in Kirgizia, in Uzbekistan, in the Ukraine, of course, in Moscow, in Leningrad etc. The biggest group works in Krasnoyarsk at Computing Center mentioned above. Materials on this subject are constantly published there in the form of preprints. As a whole, there are more than 80 authors of the published works.

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