Issues for General Users of Validated Optimization Software

What Does the Answer Mean?

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Outline

The General Global Optimization Problem

Mathematical Description

minimize $\varphi(x)$

subject to $c_i(x) = 0, i = 1, \ldots, m_1,$

$g_i(x) \leq 0, i = 1, \ldots, m_2,$

where $\varphi : \mathbb{R}^n \to \mathbb{R}$ and $c_i, g_i : \mathbb{R}^n \to \mathbb{R}$.

Automatically Verified Global Optimization

What Do the Answers Mean?

- Automatically verified global optimization software claims to find “all solutions” without missing any.
- However, such software cannot infinitely subdivide a search region. Therefore, for some problems, portions of the region will remain unresolved.
- Furthermore, there can be confusion between whether bounds are actual bound constraints or merely the extent of a finite search region.
- These issues are a source of confusion for many users of global optimization software.
- How the software handles these issues must therefore be carefully defined and explained.
- We will give several examples from `GlobSol`.

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Making the Answer Clear

What GlobSol Can and Cannot Do

GlobSol claims to either run out of time or memory, or finish, with

- a list of interval vectors (boxes) within which there is a point at which all constraints are satisfied,
- a list of boxes whose scaled diameters are within the square root of the domain tolerances,

such that any possible global optimizing points are within one of these two lists,

if there is a global optimizer within the initial bounds.

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How Can There be No Answer

- The search region can be open. That is, the bounds on the search region can be different from bound constraints.
- Mere bounds on the search region, without other constraints, define a topologically open (and hence non-compact) domain.
- There may be *no* global optimum in a non-compact domain. (In compact domains, the optimizers might occur on the boundary.)
- `GlobSol` may return no boxes, or `GlobSol` may return disjoint sets of boxes for different tolerances. Such boxes represent different unresolved subsets of the region.

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The Example

$$\begin{aligned} &\text{minimize } x_1^2 - x_2^2 \\ &\text{subject to } -x_1 - x_2 \leq 0. \end{aligned}$$

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- This problem has no global optimizers in the open search region $x_1 \in (-1, 1)$, $x_2 \in (-1, 1)$.
- On the closed box $x_1 \in [-1, 1]$, $x_2 \in [-1, 1]$, a global optimizer occurs at $(x_1, x_2) = (0, 1)$, and the optimum value is -1.

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Behavior of our GlobSol software

- With a box tolerance of 10^{-5} and no initial approximation to an optimizer, GlobSol outputs 63 "boxes with verified feasible points," all lying along the line bounding the inequality constraint, and with best upper bound for the global optimum 2.5×10^{-11}, and no other optimizers.
- With the same box, but supplying an initial guess of $(x_1, x_2) = (0, .999)$ for a global optimizer GlobSol outputs the single box

$$(\boldsymbol{x}_1, \boldsymbol{x}_2) \subseteq [-.03163, .03163], [.9673, 1].$$

- This is a seeming contradiction, since the two output solution sets are disjoint.
- This is not actually a contradiction, since GlobSol's output is only supposed to *contain* the set of solutions, and the solution set in this case is empty.

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A Related Problem

Compactifying the Feasible Set

- Suppose we tell `GlobSol` that the boundary of the box $([-1,1],[-1,1])$ represents bound constraints for the problem.
- In that case, we won't supply `GlobSol` with an initial guess for the problem.
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- These examples and other information can be found in a new draft `GlobSol` User Guide (submitted upon invitation to a special issue of *Optimization Methods and Software*).
- This User Guide has been uploaded to DROPS.
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Further Information on GlobSol

- GlobSol is portable (using standard Fortran and simulated directed rounding).
- On average, about 1 user per day has downloaded GlobSol over the past decade.
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Credits

- Besides this author, many others, including this author's students, George Corliss, and George Corliss' students, have contributed to `GlobSol`.
- We wish to thank Sun Microsystems for support during an important stage of `GlobSol`'s development.
- We wish to thank our numerous colleagues who have used `GlobSol` in their work and have given us valuable feedback.