

First Exam

Thursday, June 16, 2005

This exam is closed book. Make sure your name is on all pages. You should put your work on your own paper, and you may keep this exam sheet upon leaving. Be sure to check your work carefully, and to show intermediate computations in a logical presentation. Each question is worth 15 points, and 10 points are free.

Consider the system of equations

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 3 \\x_1 - x_2 + x_3 - x_4 &= 3 \\2x_1 + 2x_3 &= 6\end{aligned}\tag{1}$$

1. Write down the augmented matrix for the system of equations (1).
2. Put this matrix into reduced row-echelon form.
3. What is the rank of this matrix?
4. Write down the solution of the system in terms of spanning vectors and a translation vector (that is, in parametric form).
5. Does the solution represent a point, line, plane, or some higher-dimensional object?
6. Write down a matrix A and a vector b such that $Ax = b$ corresponds to the system (1), where $x = [x_1, x_2, x_3, x_4]^T$.