1. $\|(1,3,-\sqrt{6})\| = \sqrt{1^2+3^2+6} = \sqrt{16} = 4.$

$$u = \frac{1}{4}\begin{bmatrix}1\\3\\-\sqrt{6}\end{bmatrix} = \boxed{\begin{bmatrix}1/4\\3/4\\-\sqrt{6}/4\end{bmatrix}}$$

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2. A unit vector in the direction of a is $\frac{1}{\|a\|}a = \tilde{a}$

$\|a\| = \sqrt{3^2+4^2} = 5$, so $\tilde{a} = (3/5, 0, -4/5)$.

The norm of the orthogonal projection v is such that

$$\|v\| = |u\circ\tilde{a}| = |(1,2,3)\circ(3/5,0,-4/5)| = \left|\frac{3}{5}-\frac{12}{5}\right| = \boxed{\frac{9}{5}}$$

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3. $$v = (u\circ\tilde{a})\tilde{a} = \frac{-9}{5}\left(\frac{3}{5},0,\frac{-4}{5}\right) = \boxed{\left(\frac{-27}{25}, 0\ \frac{36}{25}\right) = v}$$

$$w = u - v = (1,2,3) - \left(\frac{-27}{25}, 0, \frac{36}{25}\right) = \left(\frac{25+27}{25}, 2, \frac{75-36}{25}\right)$$

$$= \boxed{\left(\frac{52}{25}, 2, \frac{39}{25}\right) = w}$$

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4. a displacement vector is: $\begin{bmatrix}4\\5\\6\end{bmatrix} - \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}3\\3\\3\end{bmatrix}$.

Thus $\boxed{\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix} + t\begin{bmatrix}3\\3\\3\end{bmatrix}}$.

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5. $$\begin{bmatrix}4 & -2 & 6\\-2 & 1 & -3\end{bmatrix} \sim \begin{bmatrix}1 & -1/2 & 3/2\\-2 & 1 & 3\end{bmatrix} \sim \begin{bmatrix}1 & -1/2 & 3/2\\0 & 0 & 0\end{bmatrix}$$

Thus, solutions to $Ax=0$ are of the form:

$x_1 = \frac{1}{2}x_2 - \frac{3}{2}x_3$, that is, $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}1/2x_2 - 3/2x_3\\x_2\\x_3\end{bmatrix}$

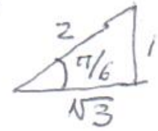
so the solutions are of the form

$t\begin{bmatrix}1/2\\1\\0\end{bmatrix} + u\begin{bmatrix}-3/2\\0\\1\end{bmatrix}$, so a basis for the null space

is $\boxed{\left\{\begin{bmatrix}1/2\\1\\0\end{bmatrix}, \begin{bmatrix}-3/2\\0\\1\end{bmatrix}\right\}}$.

(6) We will use a rotation $P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

For $ax^2 + bxy + cy^2 = 1$, we determined the required angle to have $\tan(2\theta) = b/(a-c)$.



Thus $\tan(2\theta) = \sqrt{3}/(1-0) = \sqrt{3}$, so $2\theta = \pi/3$, so $\theta = \pi/6$, and $P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$. $\begin{bmatrix} x \\ y \end{bmatrix} = P \begin{bmatrix} x' \\ y' \end{bmatrix}$

(b) Plugging in $x = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y'$, $y = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$ and simplifying:

$$\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)^2 + \sqrt{3}\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)$$

$$= \frac{3}{4}(x')^2 - \frac{1}{4}(y')^2 + \frac{\sqrt{3}}{2}x'y' + \sqrt{3}\left(\frac{\sqrt{3}}{4}(x')^2 - \frac{\sqrt{3}}{4}(y')^2 + \frac{2}{4}x'y'\right)$$

$$= \frac{6}{4}(x')^2 - \frac{1}{2}(y')^2 = 1 \qquad \boxed{\therefore \frac{3}{2}(x')^2 - \frac{1}{2}(y')^2 = 1}$$