

$$\begin{aligned} \textcircled{1} \quad & \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ -1 & -1 & -2 & | & 0 & 1 & 0 \\ 1 & 1 & 4 & | & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 1 & 1 & 0 \\ 0 & -1 & 1 & | & -1 & 0 & 1 \end{bmatrix} \\ & \sim \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 1 & 1 & 0 \\ 0 & 0 & 2 & | & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{bmatrix} \\ & \sim \begin{bmatrix} 1 & 2 & 0 & | & 1 & -3/2 & -3/2 \\ 0 & 1 & 0 & | & 1 & 1/2 & -1/2 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & | & -1 & -5/2 & -1/2 \\ 0 & 1 & 0 & | & 1 & 1/2 & -1/2 \\ 0 & 0 & 1 & | & 0 & 1/2 & 1/2 \end{bmatrix} \end{aligned}$$

$$\textcircled{b} \quad A^{-1}A = \begin{bmatrix} -1 & -5/2 & -1/2 \\ 1 & 1/2 & -1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\textcircled{c} \quad X = A^{-1}b = \begin{bmatrix} -1 & -5/2 & -1/2 \\ 1 & 1/2 & -1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -9/2 \\ -1/2 \\ 3/2 \end{bmatrix}$$

$$x_1 = -9/2, x_2 = -1/2, x_3 = 3/2$$

$$\textcircled{d} \quad Ax = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} -9/2 \\ -1/2 \\ 3/2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & -4 & -1 \\ 2 & -3 & 1 \\ 3 & -2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$$

$$\textcircled{3} \quad \begin{bmatrix} 1 & 2 & 3 & | & -1 \\ -1 & -1 & -2 & | & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & | & -1 \\ 0 & 1 & 1 & | & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & | & -3 \\ 0 & 1 & 1 & | & 1 \end{bmatrix}$$

$$\begin{cases} x_1 = -3 - t \\ x_2 = 1 - t \\ x_3 = t \end{cases}$$

$$\textcircled{4} \quad A A^T = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 14 & -9 \\ -9 & 6 \end{bmatrix}.$$

$$A^T A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 5 & 8 \\ 5 & 8 & 13 \end{bmatrix}$$