Math. 362-02
Fall, 2015
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Final Exam

Tuesday, December 8, 2015

8:00AM to 10:30AM, MDD 208

This exam is closed book, but you may use calculators. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Each entire problem is worth 16 points, and 4 points are free. You may leave when you finish, and you may keep this sheet with the questions.

1. Suppose we have a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$, such that

$$T(e_1) \;=\; (1,2),\quad T(e_2) \;=\; (-1,0),\quad T(e_3) \;=\; (3,2),\text{ and}\quad T(e_4) \;=\; (-3,2).$$

Write down a matrix A such that, for $x \in \mathbb{R}^4$, $T(x) = Ax$.

2. Compute the determinants of the following matrices; simplify your answer so it is of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where the a_i are numbers, and show your work. (You need not list terms whose coefficient $a_i = 0$.)

$$\text{(a)} \begin{bmatrix} 0 & x & 1 & 0 \\ 1 & 0 & x & 1 \\ 0 & 0 & 1 & x \\ x & 0 & 0 & 1 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{bmatrix}$$

3. Find the norm of the orthogonal projection of $u = (1,1,1)$ onto $a = (3,0,-4)$.

4. Find the orthogonal projection of $u = (1,1,1)$ onto $a = (3,0,-4)$.

5. Compute all eigenvalues and corresponding eigenvectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also, answer the following questions:

(a) Are there multiple eigenvalues?

(b) Does the set of all eigenvectors span $\mathbb{R}^4$?

6. Consider the matrix A, with

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 5 \end{bmatrix}.$$

(a) Compute the reduced row echelon form for A.

(b) List the leading variables and free variables in the reduced row echelon form.

(c) Find a basis for the row space of A.

(d) State the dimension of the row space of A.

(e) Find a basis for the null space of A.

(f) State the dimension of the null space of A.