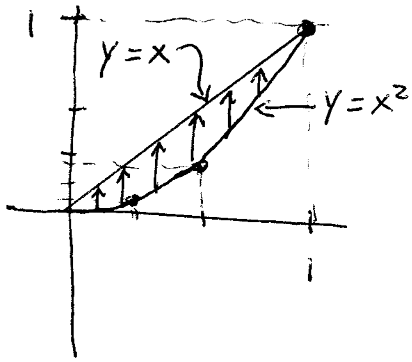
1 a



b
$$\int_{x=0}^{1}\int_{y=x^2}^{x} x\,y\,dy\,dx = \int_{x=0}^{1} x\left[\frac{y^2}{2}\right]\Big|_{y=x^2}^{x} dx$$

$$= \frac{1}{2}\int_{x=0}^{1} x\left[x^2 - x^4\right]dx = \frac{1}{2}\int_0^1 x^3 - x^5\,dx = \frac{1}{2}\left[\frac{1}{4}-\frac{1}{6}\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}-\frac{1}{3}\right] = \boxed{\frac{1}{24}}$$

2
$$\int_{x=0}^{\pi/2}\int_{y=0}^{\pi/2}\int_{z=0}^{\pi/2} \sin(x+y+z)\,dz\,dy\,dx$$

$$= \int_{x=0}^{\pi/2}\int_{y=0}^{\pi/2} -\cos(x+y+z)\Big|_{z=0}^{\pi/2} dy\,dx = \int_0^{\pi/2}\int_0^{\pi/2} -\cos(x+y+\pi/2)+\cos(x+y)\,dy\,dx$$

$$= \int_0^{\pi/2}\left[-\sin(x+y+\pi/2)+\sin(x+y)\right]\Big|_{y=0}^{\pi/2} dx = \int_0^{\pi/2} -\sin(x+\pi)+\sin(x)+\sin(x+\pi/2)-\sin(x)\,dx$$

$$= \int_0^{\pi/2} \sin(x+\pi/2)-\sin(x+\pi)\,dx = \left[-\cos(x+\pi/2)+\cos(x+\pi)\right]\Big|_{x=0}^{\pi/2}$$

$$= -\cos(\pi)+\cos(3\pi/2)+\cos(\pi/2)-\cos(\pi) = \boxed{2}$$

3 We use cylindrical coordinates.

$$\iiint_{\mathcal{V}} f\,dV = \int_{\theta=0}^{2\pi}\int_{z=0}^{1}\int_{r=0}^{1}(r^2+z^2)\,r\,dr\,dz\,d\theta$$

$$= \left[\int_{\theta=0}^{2\pi} d\theta\right]\int_{z=0}^{1}\int_{r=0}^{1} r^3 + rz^2\,dr\,dz = 2\pi\int_{z=0}^{1} \frac{1}{4}+\frac{1}{2}z^2\,dz$$

$$= 2\pi\left(\frac{1}{4}+\frac{1}{6}\right) = \pi\left(\frac{1}{2}+\frac{1}{3}\right) = \boxed{\frac{5\pi}{6}}$$

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4 We will use spherical coordinates.

$$\iiint_{\mathcal{V}} f\,dV = \int_{\rho=0}^{1}\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi}\rho^2(\rho^2\sin\varphi)\,d\varphi\,d\theta\,d\rho$$

$$= \left[\int_{\theta=0}^{2\pi} d\theta\right]\left[\int_{\varphi=0}^{\pi}\sin\varphi\,d\varphi\right]\left[\int_{\rho=0}^{1}\rho^4\,d\rho\right]$$

$$= [2\pi]\,[2]\left[\frac{1}{5}\right] = \boxed{\frac{4\pi}{5}}$$