Eldon Hansen

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INTERVAL ARITHMETIC PROGRAMS AND REFERENCES, OCTOBER 1965

A system of "Interval Arithmetic" (also known as "Range Arithmetic"), guaranteed to bound the errors on a set of computations, has been developed by Dr. Ramon E. Moore and others over the past several years. Work was done at Stanford University, Palo Alto, California, and at the Research Laboratories of the Lockheed Missiles and Space Company, Palo Alto, California.

Two areas of application utilizing "Interval Arithmetic" in which work is known to have been done so far are as follows:
1. The solution of systems of ordinary differential equations utilizing "Interval Arithmetic".
2. Matrix computations using "Interval Arithmetic".

A set of references concerned with the arithmetic system and/or its use in one of these two fields is given below.

Research in this differential equations solving area is currently being continued by Dr. Moore at the University of Wisconsin. Research on methods of application of "Interval Arithmetic" to matrix inversion and matrix computations is being carried on by Dr. Eldon R. Hansen at the Palo Alto Research Laboratories of Lockheed Missiles and Space Company (LMSC).

Inquiries beyond the scope of the reference materials, or regarding current activities, should be addressed to the respective research mathematician concerned with the field in question, as indicated below:

Field: Differential Equations Solution utilizing Interval Arithmetic; or Interval Arithmetic theory:
Address:

> Dr. Ramon E. Moore
> Mathematics Research Center
> University of Wisconsin
> Madison, Wisconsin, U.S.A.

Field: Matrix Computations using Interval Arithmetic:
Address:

> Dr. Eldon R. Hansen
> Palo Alto Research Laboratories, Dept. 52-40
> Lockheed Missiles and Space Company
> 3251 Hanover Street
> Palo Alto, California

Computer programs that have been written in these various areas and are known to be either partly or fully operational include the following:

A. Systems of interval arithmetic subroutines:

   1. Interval Arithmetic system of subroutines, written in FAP for the 7094 computer, at LMSC, currently operational. Most of the basic arithmetic operations and some logical operations are available in both single and double precision forms. The standard functions (sin, cos, log, etc.) are not currently available at LMSC, although some work has been done on these. For information contact:

   > Mr. William M. Lowney, Department 59-32
   > Palo Alto Research Laboratories
   > Lockheed Missiles and Space Company
   > 3251 Hanover Street
   > Palo Alto, California, U.S.A.

   2. An Interval Arithmetic system was developed at the University of Wisconsin in 1964-65. For information on the computer and status contact Dr. Ramon E. Moore (address given earlier).

B. Solutions of systems of ordinary differential equations utilizing Interval Arithmetic:

   1. Program "DIFEQ", Version I, was developed at LMSC in 1963-64. It was written in a combination of FORTRAN II and FAP for the 7094. DIFEQ Version I accepts an English-like set of statements, representing its equations, as input. DIFEQ Version I is an operational system and has been submitted to the SHARE organization. This program (in binary form) and its writeup can be obtained from SHARE. The SHARE name of this program is ML MDJS, and the SHARE distribution number is SDA3210. The source deck is not available. For copies of this program contact:

   > SHARE Distribution Agency
   > DP Program Information Department
   > IBM Corporation
   > 112 East Post Road
   > White Plains, New York, U.S.A.   10601

   Give the program name (ML MDJS), title (DIFEQ integration routine using interval arithmetic), and SHARE distribution number (SDA3210), and indicate which of the decks or writeups available for this program are required.

2. Program DIFEQ, Version II, which accepts its input equations in algebraic form, was developed at LMSC in 1964-65. It is also written in a combination of FORTRAN II and FAP for the IBM 7094 computer. The system is operational, and includes some features not available in DIFEQ, Version I. A complete source deck and/or binary decks are available; however the writeup is not yet available and is not expected to be out for some time. For information contact Mr. W. M. Lowney (address given above).

3. A new version of the Differential Equations solving system (presumably in FORTRAN) was written at the University of Wisconsin in 1965 under the auspices of Dr. Moore. The inclusion of new coordinate transformation strategies within this system was anticipated; it is not known whether this has been completed. For information contact Dr. Moore (address given earlier).

C. Matrix computations utilizing Interval Arithmetic:

1. A matrix inversion program was developed by Roberta Smith, under the research direction of Dr. E. R. Hansen, at LMSC in 1965. Written in FORTRAN II and FAP for the 7094, this program is operational. For information contact Dr. Hansen (address given earlier). A writeup will be provided on request.

2. The matrix inversion methods of Dr. Hansen were re-programmed at the University of Wisconsin in 1965, under the auspices of Dr. Moore. This program is also known to be operational; it is presumed to be in FORTRAN.

3. A routine for solving systems of linear equations (of the form AX=B) utilizing Interval Arithmetic was developed at LMSC by Hansen and Smith in 1965. The language is a combination of FORTRAN II and FAP. No theory writeup is available; this will later be published in the Journal of the Society of Industrial and Applied Mathematics, as Part II of a series by Dr. Hansen. A user's writeup will be made available on request.

4. Programming is underway on an eigenvalue-eigenvector routine utilizing Interval Arithmetic; this work is being done at LMSC by Hansen and Smith. A theoretical paper on this subject will later be published as Part III of the series by Dr. Hansen in the Journal of the Society of Industrial and Applied Mathematics. The language of this program will be a combination of FORTRAN II and FAP.

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A list of new or significant associated references follows.
Many more references can be obtained from the Reference sections
of these works. Certain articles to be published later have already
been mentioned above.

1. Thesis of Dr. R. E. Moore, at Stanford University, Palo
Alto, California, 1962. Subject: Interval Arithmetic and
Automatic Error Analysis in Digital Computing. This thesis
can be obtained (at some cost) in microfilm or printed form
from:

> University Microfilms Corp.
> 515 N. First Street
> Ann Arbor, Michigan

This thesis was also published by the Applied Mathematics and
Statistics Laboratories, Stanford University, Stanford,
California, as Technical Report No. 25 (November 15, 1962)
under an Office of Naval Research contract.

2. Authorship on a new book by Dr. R. E. Moore on Interval
Arithmetic is now complete, and publication is imminent.
For information contact Dr. Moore (address given earlier).

3. Two articles by Dr. R. E. Moore on associated subjects
have been published recently in a two-volume set of collected
works by Wiley & Sons. The books are entitled Error in
Digital Computation, edited by L. B. Rall. Volumes I and II
are available; each contains one article by Dr. Moore.

4. An article by Dr. E. R. Hansen entitled "Interval
Arithmetic in Matrix Computations. Part I" will be
published in the next issue (this should be Volume 2,
Number 2) of the Journal of the Society of Industrial and
Applied Mathematics (Series B: Numerical Analysis). This
article is concerned with matrix inversion. The Journal
is published by SIAM, Philadelphia, Pa.

A number of LMSC reports on associated subjects have also
been published over past years.