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Linear Interval Equations: Computing Sufficiently Accurate Enclosures is NP-Hard

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For a system of linear interval equations

$$A^I x = b^I \tag{1}$$

(A^I square), **enclosure** is defined as an interval vector $[\underline{y}, \overline{y}]$ satisfying

$$X \subseteq [\underline{y}, \overline{y}]$$

where X is the solution set:

$$X = \{x;\, Ax = b \text{ for some } A \in A^I, B \in B^I\}.$$

If A^I is regular, then there exists the narriwest enclosure $[\underline{x}, \overline{x}]$ given by $\underline{x}_i = \min_X x_i$, $\overline{x}_i = \max_X x_i$ for each i. Computing $[\underline{x}, \overline{x}]$ was proved to be NP-hard [1]. But it turns out that the same is true for computing "sufficiently accurate" enclosures:

Theorem 1 *Suppose there exists a polynomial-time algorithm which for each* **strongly** *regular $n \times n$ matrix A^I and each b^I (both with rational bounds) computes a rational enclosure $[\underline{y}, \overline{y}]$ for (1) satisfying*

$$\overline{x}_i \le \overline{y}_i \le \overline{x}_i + \frac{1}{32n^4} \tag{2}$$

for each i. Then **P=NP**.

Comments

A^I is called strongly regular if $\rho(|A^{-1}\Delta) < 1$ (a well-known sufficient regularity condition).

P and NP are well-known complexity classes. The conjecture that P≠NP, although unproved, is widely believed to be true.

Hence, the problem of computing sufficiently accurate enclosures is by by far more **difficult** than previously believed: an existence of a polynomial-time algorithm yielding the accuracy (2) would imply polynomial-time solvability of all problems in the calss NP, thereby making an enormous breakthrough in theoretical computer science.

References

[1] J. Rohn, V. Kreinovich, "Computing exact componentwise bounds on solutions of linear systems with interval data is NP-hard", *SIAM Journal on Matrix Analysis and Applications (SIMAX),* 1995, Vol. 16, No. 2 (to appear).

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