From Intervals to Fuzzy Truth-Values: Adding Flexibility to Reasoning Under Uncertainty

Ramon Lopez de Mantaras

Early uncertainty management systems like certainty factors [13] and probability based approaches [3, 7], were dealing only with one single real value chosen within a predefined range to represent uncertainty. These approaches had serious problems, in particular it has been argued that human numerical estimations of uncertainty have little interobserver and intraobserver consistency [5, 10]. Even worse, most experts are unable to make a fair estimation of the inaccuracy of their judgments, making far larger estimation errors than the boundaries accepted by them as feasible [8].

A first step out of this problem, that added some flexibility to the uncertainty representation problem, was proposed in several interval valued approaches, some generalizing probability based approaches, like Quinlan's consistency and plausibility masures or Dempster-Shafer belief and plausibility measures [12]; and others outside the probabilistic setting like the approach of the possibility and necessity measures [6] based on the notion of fuzzy set [14]. Besides making easier for the expert to preserve consistencies, these two-valued approaches allow for an explicit representation of ignorance a concept not easily representable with single-valued approaches; for example in single valued probability based approaches the lack of certainty about an event implies the certainty of its negation even in situations where the poverty of information would suggest that the best we could say is that we are uncertain about both, the event and its negation i.e. we would like to explicitely express our ignorance.

Even though the two-valued (interval valued) approaches have been a valuable step, they still lack the capability of dealing with imprecision, a very important source of uncertainty mainly in artificial intelligence and particularly in expert systems.

I. Uncertainty and Imprecision

At this point it may be interesting to point out the differences and relationship between uncertainty and imprecision. For example, given the imprecise information: *the temperature of the reactor is over 200 degrees*, the proposition: *the temperature of the reactor is exactly 300 degrees* is uncertain but very precise. On the other hand, if the known information is very precise, like for example: *the temperature of the reactor is 250 degrees*, then there is no room for uncertainty, that is, we will always be able to say whether a proposition is true or false. However, if the known information contains terms with a vague meaning such as: *the temperature of the reactor is high*, then, in general the only thing we will be able to say is to what degree a given temperature is a high temperature. This degree is precisely the degree of membership of the given temperature to the fuzzy set HIGH TEMPERATURE.

II. Posibility Theory

It is worth noting that a given membership value has two different meanings. On the one hand, it denotes the degree of truth of a proposition about a variable when the exact value of this variable is known (for example, let us assume that 0.8 is the degree of membership of the temperature value 600 degrees in the fuzzy set HIGH TEMPERATURE, then 0.8 is the degree of truth of the proposition *high temperature* for a temperature value of exactly 600 degrees). On the other hand, a membership value also denotes the degree of certainty about the fact that the variable takes a concrete value (i.e. 0.8 is the degree of certainty of the temperature 600 degrees knowing that the temperature is high). This second interpretation is the one associated with the concept of possibility distribution [15]. Possibility theory is at the basis of *Possibilistic Logic.*

III. Possibilistic Logic, Many-Valued Logic and Fuzzy Logic

Possibilistic logic and Many-Valued logic are particular cases of fuzzy logic. In the case of possibilistic logic, we are interested in the evaluation of the truth of a crisp (non vague) proposition given some vague information like in the temperature example above. When we are interested in the truth evaluation of a vague proposition given a crisp information, we are in the setting of many-valued logics.

The author is with IIIA–Artificial Intelligence Research Institute, CSIC–Spanish Research Council, Campus Universitat Autonoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.

In the more general case in which we evaluate the truth of a vague proposition given a vague information we are in the setting of fuzzy logic. In that case the result of the evaluation is a fuzzy set called fuzzy truth-value. Fuzzy truth-values allow to integrate the vagueness as well as the uncertainty resulting from imprecise information within the same framework. Fuzzy truth-values are fuzzy subsets of [0,1] taking values on [0,1] that are referred to by means of linguistic terms such as: *absolutely false, very false, false, fairly false,..., very true, absolutely true*. They provide a very flexible way of representing uncertainty and reasoning under uncertainty.

IV. Reasoning Within Fuzzy Truth-Valued Logics

A fuzzy truth-valued logic is a logic whose truth-values are fuzzy truth-values (i.e. linguistic values). In this logic, propositions are fuzzy truth-qualified and the generalized modus ponens inference rule is as follows: Given a fuzzy truth-qualification T1 of a rule *If X is A then Y is B* and a fuzzy truth-qualification T2 of the fact *X is A*, one can deduce a fuzzy truth-qualification T3 for the conclusion *Y is B*, where T3 is computed from T1 and T2 (see [11] for details).

We have developped an expert systems building tool based on this approach and several large scale expert systems have been developped mainly in two medical applications [9, 4] the results obtained are remarkable since in all the applications the experts felt extremely confortable with the possibility of expressing their uncertainty by menas of linguistic truth-values. Furthermore the performance of the espert systems was compared, by mans of a cluster analysis, with several physicians (6 in the case of pneumonia diagnosis and 12 in the case of rheumatology) and in both cases the expert systems clustered with the best physicians.

V. Conclusions

Looking at the chronology of the approaches to deal with uncertainty we have noticed a sustained tendency to increase its flexibility in order to avoid problems of inconsistency. The most flexible approach to date is that of the fuzzy truth-values which is based on fuzzy logic. We believe that the fuzzy truth-values approach to reasoning is the most powerful idea within fuzzy logic both from the theoretical and the practical point of view.

References

[1] R. E. Moore, **Interval analysis**, Prentice Hall, Englewood Cliffs, NJ, 1966.

[2] A. B. Author, "Bad applications of interval computations", *International Journal of Bad Applications*, 1994, Vol. 8, pp. 666–669.

[3] J. B. Adams, "A probability model of medical reasoning and the MYCIN model", *Mathematical Biosciences*, 1976, Vol. 32, pp. 177–186.

[4] M. Belmonte-Serrano, C. Sierra, and R. Lopez de Mantaras, "RENOIR : An expert system using fuzzy logic for rheumatology diagnosis", *International Journal of Intelligent Systems*, 1994, Vol. 9, pp. 985–1000.

[5] R. Beyth-Marom, "How probable is probable (a numerical taxonomy translation of verbal probability expressions)," *Journal of Forecasting*, 1982, Vol. 1, pp. 257–269.

[6] D. Dubois and H. Prade, **Possibility Theory**, Plenum Press, N.Y., 1988.

[7] R. O. Duda, P. E. Hart, and N. J. Nilsson, "Subjective Bayesian methods for rule-based inference systems", in *Proceedings of the AFIPS National Computer Conference*, 47, 1976, p. 1075–1082.

[8] C. Freksa and R. Lopez de Mantaras, "A learning system for linguistic caregorization of soft observations", *Proc. Colloque Association Recherche Cognitive, Orsay*, 1984, pp. 331–345.

[9] L. Godo, R. Lopez de Mantaras, C. Sierra, and A. Verdaguer, "MILORD: The architecture and the management of linguistically expressed uncertainty", *International Journal of Intelligent Systems*, 1989, Vol. 4, pp. 471–501.

[10] E. Horvitz and D. Heckerman, "The inconsistent use of measures of uncertainty in artificial intelligence research", in Kanal and Lemmer (eds.), **Uncertainty in Artificial Intelligence**, North-Holland, 1986, pp. 137–151.

[11] R. Lopez de Mantaras, *Approximate Reasoning Models*, Ellis Horwood Publ., 1990.

[12] G. Shafer, **A Mathematical Theory of Evidence**, Priceton University Press, 1976.

[13] E. H. Shortliffe and B. G. Buchanan, "A model of inexact reasoning in medicine", *Mathematical Biosciences*, 1975, Vol. 23, pp. 351–379.

[14] L. A. Zadeh, "Fuzzy Sets", *Information and Control*, 1965, Vol. 8, 338–353.

[15] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility", *Fuzzy Sets and Systems*, 1978, Vol. 1, pp. 3–28.