Equations Of Physics Become Consistent If We Take Measurement Uncertainty Into Consideration

A. B. Korlyukov[1] and V. Kreinovich[2]

I. Classical Physics: Idealized Description

In classical physics, the world consists of *particles* and *fields*. Therefore, to describe the current state of the world, we must describe:

- the current positions $\vec{x}^{(1)}, \vec{x}^{(2)}, \ldots$ and velocities $\vec{v}^{(1)}, \vec{v}^{(2)}, \ldots$ of all the particles, and
- the current values $f(\vec{x})$ of all the fields f in all spatial points $\vec{x}$.

Physical equations describe how a state changes with time. So, if we can measure a current state, we will be able to predict the future states (this idea is called *Laplace determinism*).

II. In Classical Physical, We Can (Potentially) Measure With Arbitrary Accuracy

Of course, we cannot measure the state with absolute accuracy, but nothing in classical physics prevents us from knowing it with better and better accuracy. For example, a realistic field probe has non-zero thinness, and thus, it measures not the value of the field in one spatial point, but an average over all the points covered by this probe. However, we can imagine thinner and thinner probes that can compute the averages over the smaller and smaller areas surrounding a given point $\vec{x}$, and therefore, produce the measurement results that are closer and closer to the desired value $f(\vec{x})$.

III. Quantum Physics: Uncertainty Principle

The major change that has been introduced by quantum physics is related to the so-called *uncertainty principle*. According to this principle, the better spatial accuracy we want to achieve, the more energy we have to pump into the system. If we pump lots of energy into a small area of space, then we will sure disturb the very process that we want to measure.

The authors are from [1]Grodno, Belarus, and from the [2]Department of Computer Science, University of Texas at El Paso, El Paso, TX 79968, email vladik@cs.utep.edu. V. Kreinovich is thankful to A. Neumaier for inspiring discussions. This work was partially supported by NSF grant No. CDA-9015006 and NASA Research Grant No. 9-757.

As a result, if we use thinner and thinner field probes in the vicinity of a given spatial point $\vec{x}$, we are measuring the more and more disturbed field. Therefore, if we want to follow the idea of classical mechanics and define the "true" value $f(\vec{x})$ of a physical field f at a point $\vec{x}$ as a limit of the values that we get when we apply thinner and thinner probes, then, instead of the sequence of values that converge to a physically meaningful finite limit, we get a divergent process that does not have a finite limit.

IV. Divergence Problem

This phenomenon leads to a serious problem of quantum physics: Indeed, quantum equations are usually obtained by modifying the classical ones, and therefore, these equations are usually formulated in terms of point values of the fields. If we try to integrate these differential equations, we thus get senseless infinite results for $f(\vec{x})$ (called *divergences*).

V. The Existing Solutions to This Problem

There exist several semi-formal tricks that help to overcome this problem and come up with finite predictions for experimentally observable quantities. It would be nice, however, to use some physically valid and mathematically idea instead of the heuristic tricks.

VI. Our Main Idea

From our viewpoint, however, the most reasonable approach will be not to use any mathematical formal tricks, but to nip this problem in the bud by avoiding physically meaningless point values of the fields, and thus, to describe the fields in terms of observable quantities only.

In other words, we want to describe a physical field by the values that are measured by different field probes.

VII. This Idea Leads to 9-Dimensional Space

As we have already mentioned, a probe cannot be located exactly in a point $\vec{x}$. No matter how accurately we locate it, there is always an inevitable location error. This error is unpredictable and can be, thus, viewed as random.

There can be many different sources of a location error. If we have one prevailing error source, depending on one or few parameters, then usually, we can compensate for this particular error. So, if a field probe has been carefully designed, all dominating errors have been eliminated, and the remaining error is the result of the joint action of several small independent error components of similar size. According to the limit theorems of mathematical statistics (see, e.g., [3], [1], [6], pp. 2.17, 6.5, 9.8, and references therein), this error is thus distributed according to the Gaussian law, with the density

$$\rho(x_1, x_2, x_3) = const \cdot \exp(-\frac{1}{2}\sum_{ij} A_{ij}\Delta x_i \Delta x_j),$$

where:

- $\Delta x_i = x_i - a_i$ is the location error, i.e., the difference between the actual coordinates x_i and the desired coordinates a_i, and
- A_{ij} is a symmetric matrix that describes how small the locations errors are.

So, to describe a probe, we need to know 9 parameters:

- 3 parameters a_i that describe the desired location, and
- 6 parameters A_{ij} that characterize the location error.

Let us denote this set of parameters by

$$\vec{p} = (a_1, a_2, a_3, A_{11}, A_{12}, A_{13}, A_{22}, A_{23}, A_{33}).$$

To describe the quantum field, we need to know the values measured by every possible probe. In other words, for each set of 9 parameters $\vec{p}$, we must know the "average" value $\bar{f}(\vec{p})$ of the field that is measured by the corresponding probe. So:

- if we describe a classical field in observable terms, it is sufficient to describe a function of 3 spatial variables.
- To describe a quantum field, we need a function of 9 variables.

Mathematically, it is equivalent to saying that we consider arbitrary fields in a *9-dimensional space*.

VIII. 9-D space Means 10-D Space-Time

In the above calculations, we considered the state of the field at a given moment of time. If we take time into consideration, we will arrive at a conclusion that to describe how a field evolves, we need a function $\bar{f}(\vec{p}, t)$ of 10 parameters (time is an additional parameter). Mathematically, this is equivalent to saying that we have to consider a *10-dimensional* space-time.

IX. In 10-D Space-Time, Quantum Field Theory Is Consistent

It is interesting to mention that 10 is indeed the smallest dimension of space-time for which a consistent ("*renormalizable*") theory is possible (see, e.g., [2, 5]).

X. Connection Between Our Idea And String Theory

Another proof that the above idea makes sense is as follows:

- In our interpretation, instead of points in space, we consider distributed space areas.
- The mathematical formalism that underlies the correspondent consistent theory is a so-called *string theory*, in which the basic objects are also not point particle, but spatially distributed objects ("strings").

So, our idea can be viewed as a (natural) generalization of a string theory: instead of 1-D objects, we have 3-D ones.

Comment. Another application of interval approach to foundations of physics is described in [4].

References

[1] T. V. Arak and A. Yu. Zaitsev, **Uniform limit theorems for sums of independent random variables**, (Proceedings of the Steklov Institute of Mathematics, Vol. 174), American Mathematical Society, Providence, RI, 1988.

[2] L. Brink and M. Henneaux, **Principles of string theory**, Plenum Press, N.Y., 1988.

[3] B. V. Gnedenko and A. N. Kolmogorov, *Limit distributions for sums of independent random variables*, Addison-Wesley, Cambridge, 1954.

[4] A. V. Korlyukov, "Introduction to interval field theory", *Proceedings of the International Conference on Interval and Stochastic Methods in Science and Engineering INTERVAL'92*, Moscow, 1992, Vol. 1, pp. 71–73 (in Russian; English abstract Vol. 2, p. 46).

[5] W. Siegel, **Introduction to String Filed Theory**, World Scientific, Singapore, 1988.

[6] H. M. Wardsworth, Jr. (editor), **Handbook of statistical methods for engineers and scientists**, McGraw-Hill Publishing Co., N.Y., 1990.