Estimating Uncertainties for Geophysical Tomography

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We present new techniques for evaluating the uncertainties associated with geophysical tomographic inversion problems including estimation of data errors, model errors, the information density matrix and total solution uncertainty. Our inversion method uses the conjugate gradient technique [5], incorporating a priori estimates of data and model uncertainties to stabilize the solution. We apply this inversion method to travel time data collected in a cross well seismic experiment [3] and to data collected during the experimental freezing and thawing of a soil filled tank [6].

Seismic tomography involves using the travel times of seismic (sonic) waves to reconstruct the velocity or inverse velocity (slowness) distribution of the material between sources and receivers. The velocities or slownesses provide estimates of the possible geologic formations or rock compositions that may exist between the sources and receivers; factors important to groundwater, mining, petroleum, and geotechnical studies.

The "standard" estimate of image quality from travel times test only limited contributions to uncertainty and are qualitative (i.e. [2]). The "standard" techniques estimate the solution variance by calculating artificial travel times for a synthetic model. Gaussian noise is added to the travel times. Next, the noisy travel times are inverted using the same ray paths as the original (observed) data set. A comparison of the tomographic image from noisy data with the image obtained from the observed data provides a qualitative measure of the image quality. Resolution is estimated by constructing a slowness model, such as a checkerboard pattern, calculating travel times through the model using the same ray paths as the observed data set, and inverting the travel times to see if the slowness image matches the initial slowness model. These estimates of resolution and variance are generally done for only one test case (one set of noisy artificial data, one checkerboard slowness model), and assume the travel times and slownesses are known perfectly in the inversion of the original data.

Our approach differs from the "standard" technique in the following ways:

1) We allow for model and data error by assigning a priori uncertainties to the travel times and using an initial starting model of slownesses that also has associated uncertainty values. These uncertainties are based on other geological and geophysical information. The inversion approach uses the conjugate gradient technique where a priori information can be included to increase stability and speed of convergence.
2) We obtain a quantitative estimate of the information density matrix [4] by repeatedly adding noise to the observed travel times and inverting for slowness images. The variations between the slowness images gives the covariance of each velocity model block.
3) We estimate the total uncertainty of the solution by adding noise to both the data and the starting slowness model. We then conduct multiple inversions of the data and slowness models with different noise sets.
4) We have examined the effects of error in picking raw travel time data by inverting subsets of data that have bad data points removed. In addition, we examined the distribution of data errors (difference between every observed travel time and its corresponding calculated travel time) to test whether the l^2 norm was the appropriate data norm for our inversion process.
5) Model validity was evaluated by using a variety of different starting models that assumed isotropic or anisotropic velocity distributions. The distribution of model errors (differences between each slowness value in the starting model and its corresponding slowness valued calculated from the inversion) was also determined.

Our most exhaustive studies of error estimation used a data set of travel times collected from a cross well experiment run in an oil field near Rifle, Col-

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orado [1]. The two wells were separated by 34 m and we concentrated on a depth interval between 1800 and 2100 m. A 2.2 kHz source transmitted signals at a rate of 1.5 signals/m. The receiver was held stationary for each transit of the source and then moved 1.5 m between transits. The velocity model between wells was constrained by velocity logs run in the source and receiver wells. We constructed both anisotropic and isotropic a priori starting models consistent with the velocity logs.

We found that between 10 and 15 inversions sets were adequate to accurately estimate the information density matrix and total uncertainty. Best fit to the data came from a starting model of uniform isotropic layers, although we expected an anisotropic model to fit better. It appeared that incorrect travel time picks altered much of the anisotropic information, especially in the lower part of the well where the data indicated anisotropy would be the greatest. The data norm appears to be near 2, but the model norm is close to 1, so that using an l^2 norm for the inversion process may not be appropriate. No clear relationship between the total uncertainty and known geology was observed, but vertical slowness uncertainties were much less than horizontal uncertainties.

We are still completing analysis of travel time data collected during a laboratory experiment where a tank of soil underwent freeze/thaw cycles. Exhaustive estimates of travel time errors were made during data collection. These observation errors were found to underestimate by a factor of 4 the level of errors required for consistency with a reasonable slowness model. We are still searching for the source of this inconsistency and initial results indicate ASTM procedures specifying the seismic source are inappropriate for the scale of the experiment.

The explicit use of a priori model estimates and data uncertainties in tomographic estimation problems permits identification of the source of data errors as well as providing solution uncertainty estimates based on information density and resolution. These techniques can be applied to any problem where a useful solution estimation algorithm exists.

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