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Interval and Fuzzy Approaches to Queuing Systems

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I. Queuing Models Describe Many Real-Life Phenomena

Queuing models adequately describe many real-life systems:

- *communication networks*;
- *management of the engineering team projects*;
- *transportation systems*;
- *flexible manufacturing systems*;
- *service systems*;
- etc.

II. In Many Real-Life Cases, Queuing Models Lead To Explicit Or Implicit Formulas

In many real-life cases, we can apply queuing models (e.g., M/M/1) that are *simple* in the following sense:

- either, we have an *explicit* formula

  $$y = f(x_1, ..., x_n)$$

  that describes the desired characteristics y of the queue as a function of the model's parameters $x_1, ..., x_n$, or
- we have an *implicit* equation

  $$F(y, x_1, ..., x_n) = 0$$

  from which, if we know x_i, we can easily compute y.

III. Parameters Of A Queuing Model Are Not Known Precisely

In real life, the value x_i of the parameters of the queuing model must be determined from the experimental data. Therefore, these parameters are never absolutely precise: instead of the actual values of these parameters, we know only the *intervals* $\mathbf{x}_i = [\tilde{x}_i - \Delta_i, \tilde{x}_i + \Delta_i]$ of possible values of x_i, where $\tilde{x}_i$ is an estimate, and Δ_i is an accuracy of this estimate.

Traditionally, as an estimate $\tilde{y}$ for the desired characteristic y, we take correspondingly either the value $\tilde{y} = f(\tilde{x}_1, ..., \tilde{x}_n)$, or the solution of the equation $F(\tilde{y}, \tilde{x}_1, ..., \tilde{x}_n) = 0$.

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IV. As A Result Of This Uncertainty, Estimated Values Of The Desired Characteristics Of The System May Differ From Their Actual Values

Our numerical experiments show that if we are determining the parameters x_i from a small sample, then the values Δ_i may be large, and the resulting deviation of $\tilde{y}$ from the actual value of y may also be large.

V. Interval Approach To Queuing Models

In view of the above-mentioned difference, it is better, instead of an estimate $\tilde{y}$, to provide the user with an *interval* $\mathbf{y}$ of possible values of y:

- For an *explicit* model, this interval can be computed by applying interval computations as $\mathbf{y} = f(\mathbf{x}_1, ..., \mathbf{x}_n)$.
- For an *implicit* model, as $\mathbf{y}$, we take an enclosure of the set of all solutions of the equation $F(y, \mathbf{x}_1, ..., \mathbf{x}_n) = 0$.

We have applied these methods to several real-life systems.

VI. Fuzzy Approach

In some cases, in addition to the interval $\mathbf{x}_i$, the experts can supply us with the *degrees of possibility* $d_i(x_i)$ of different values x_i from this interval. As a result, we have a *fuzzy membership* function. In this case, in addition to the interval $\mathbf{y}$, we can also compute the degrees of belief $d(y)$ of different values of y:

Namely, for every level $\alpha \in [0, 1]$, we can do the following:

- for every i, we can find the interval $\mathbf{x}_i(\alpha)$ that contains only those values $x_i \in \mathbf{x}_i$ that are possible with degree of possibility $\geq \alpha$ (i.e., for which $d_i(x_i) \geq \alpha$).
- apply the above-described interval techniques to the resulting intervals $\mathbf{x}_i(\alpha)$, and thus find the interval $\mathbf{y}(\alpha)$ of values y that are possible with this degree of possibility.

We have designed a software for these computations.

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