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On Interval Factorial Genetic Algorithm in Global Optimization

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***Abstract*— In this work, we combine three promising global optimization methods:**

- **interval arithmetic methods,**
- **experimental design, and**
- **genetic algorithms,**

into a hybrid algorithm that seems to be superior to any of these single methods in engineering applications where a good estimates of the global optimum of multimodal function is required.

- **Interval arithmetics gives us simple methods to estimate the range of functions to be optimized.**
- **Experimental design and factorial analysis helps to find the essential parameter values while**
- **Genetic algorithms adaptively guides the search.**

It was not a great surprise that the results that we obtained using this new hybrid method were superior to other similar methods. The analysis now continues to reveal the power of the proposed hybrid in real industrial applications.

Keywords: experimental design, factorial analysis, genetic algorithms, hybrid methods, interval arithmetics, global optimization, multimodal optimization.

I. Introduction

In this paper, we discuss the following simple type of global optimization problem:

$$\min f(x) = f(x_1, x_2, \ldots, x_n)$$

$$l_i \leq x_i \leq u_i, \quad i = 1, \ldots, n$$

where $f : R_n \rightarrow R$. We denote by $\mathbf{X}$ the box

$$\{x = (x_1, \ldots, x_n) \,|\, l_i \leq x_i \leq u_i, i = 1, \ldots, n\}.$$

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There are many well-known local optimization methods that can be used to solve this problem when the function f is unimodal. In the case when f is multimodal, the situation is totally different, and the problem can be extremely complex. The book by Horst and Tuy [13] contains a good review of the known global optimization methods.

A. Interval arithmetics

Interval arithmetic operations [18] are defined for ordered pairs of real numbers,[1] which represent closed intervals on the real line:

$$A = [a_1, a_2] = \{x \in R \,|\, a_1 \leq x \leq a_2\}.$$

The first element of the pair is the minimum value of the interval often denoted by $\inf(A)$, while the second element is the maximum denoted by $\sup(A)$ accordingly. Real numbers x can be identified with degenerate intervals $[x, x]$.

The interval arithmetic operators are usually defined as follows:

$$A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2],$$

$$A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1],$$

$$A \cdot B = [a_1, a_2] \cdot [b_1, b_2] =$$

$$[\min(a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2),$$

$$\max(a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2)],$$

$$A/B = [a_1, a_2]/[b_1, b_2] = [a_1, a_2] \cdot [1/-b_2, 1/b_1].$$

The most valuable property of interval arithmetics in numerical analysis and especially in optimization is its inclusion monotonicity. Let A, B, X, and Y be intervals such that $A \subseteq X$ and $B \subseteq Y$, then

$$A + B \subseteq X + Y,$$

$$A - B \subseteq X - Y,$$

$$A \cdot B \subseteq X \cdot Y,$$

$$A/B \subseteq X/Y.$$

[1] It is also possible to define interval arithmetics on complex numbers. Then intervals are circles on the complex plane [25, 26].

In general, inclusion monotonicity holds is for any rational function $F(A_1, \ldots, A_n)$: If $A_1 \subseteq X_1, \ldots, A_n \subseteq X_n$, then

$$F(A_1, \ldots, A_n) \subseteq F(X_1, \ldots, X_n)$$

for all values $X_1, \ldots, X_n$, for which F is defined [18]. In particular, if we take degenerate intervals (scalars) $x_1, \ldots, x_n$ as A_i, we conclude that if $x_i \in X_i$ for all i, then

$$f(x_1, \ldots, x_n) \in F(X_1, \ldots, X_n).$$

In other words, this means that

$$f^*(X_1, \ldots, X_n) \subseteq F(X_1, \ldots, X_n),$$

i.e., the interval extension F of a scalar function f is a more or less good upper bound of the range of f (denoted by f^*) over the given hyperbox $\mathbf{X} = X_1 \times X_2 \times \cdots \times X_n$.

The quality of the interval range approximation depends on the form of the function to be evaluated. In the special case when every variable x_i occurs exactly once and to the first power in f, the interval generalization gives the exact range f^* [19]. Chuba and Miller have shown that the error is between $O(w)$ and $O(w^2)$, where w is the width of the argument intervals [3]. A comparison of different interval methods can be found, e.g., in [1]. The so-called *centered forms*, described in Subsection A.1. , seems to be reasonably efficient in approximating f^*.

B. Experimental design and factorial analysis

A good introduction to experimental design and factorial analysis is given, e.g., in [17]. Nam-Ky Nguyen has recently described an algorithm for constructing optimal or near-optimal incomplete block designs [22].

C. Genetic algorithms

Living organisms contain huge amounts of information in the form of genetic code. Parts of the genetic coding and reproduction system are rather well known and the apparent billion years triumph of biological evolution has inspired computer scientists to create, analyze, and utilize similar systems in problems that are known to be computationally hard. This has led to the study of genetic algorithms (GA) in optimization and related fields.

One attractive feature of genetic algorithms is their simplicity, which makes it easy to implement and tailor them to practical problems. The operation of a genetic algorithm searching for an optimal solution to a given problem can be divided into the following steps:

Genetic Algorithm:

- [**initialization**] generate the initial (e.g. random) set of potential solutions;
- [**natural selection**] calculate how "far" is each solution from the goal, and take the best ones;
- [**breeding**] generate new solutions by exchanging solution parameters (crossing) between the selected good solutions, and possibly, add some noise (mutations);
- [**repetition**] if the goal is not reached, go back to the step [**breeding**].

In spite of the seemingly simple processing, the GAs are known to be good in solving some problems that are known to be hard. The simplicity, flexibility, parallelism, and the good problem solving capability were the most important facts that inspired us to experiment with GAs in order to find methods of solving robot control problems.

Some apparent good and bad properties of GAs include:

good news	*bad news*
simple	massive
parallel in nature	large storage
robust	stochastic
general	not well known

The fundamentals of genetic algorithms can be found in David Goldberg's book [10]. In addition, Lawrence Davis gives a good overview of applications [4]. He also suggests a practical procedure to attack new problems by combining genetic algorithms with existing methods into, as he calls, hybrid GA: The pure basic genetic algorithm is usually not the best possible algorithm to solve any specific problem if a good specific algorithm already exists. There is, however, a good chance to create a better algorithm by combining the best features of existing conventional and genetic algorithms.

D. Related work

Interval methods have not been used much with genetic algorithms. According to the bibliography [2] compiled by the author, only the following papers are related to intervals: [21, 20, 7, 15, 16]. Because there are few interval GA papers, one objective of this paper is to make interval arithmetics and its potential application in genetic algorithms more familiar to the GA community and *vice versa*; the author also hopes that this paper also makes GAs and their potential in optimization more familiar to the interval community. According to [2], only four papers dealing with GA have appeared in the mainstream optimization journals [6, 9, 14, 29], which is quite small number for such an inherently optimization method as GA!

The lack of papers that we have noticed for GAs/interval arithmetics application applies to GAs and factorial analysis as well. Only one reference in [2] seems to have something to do with factorial analysis and GAs: [8].

II. Methods

A. Interval arithmetics

Interval methods for optimization are based on the inclusion monotonicity of interval generalizations of scalar functions. In global optimization, branch and bound method is usually used to find the exact global optimum. The argument interval are subdivided until the optimum of given tolerance is found [11, 12].

Ratschek and Rokne have proposed the following simple interval optimization method [24, Algorithm 2 of Chapter 5]:

1. put all uninspected boxes into a list L. At the beginning, L contains only the search box X.
2. subdivide the first box of L into boxes V_1 and V_2.
3. put box V_i at the end of L unless it can not contain a global solution
4. if a stopping criterion is met, terminate, else go to step 2.

A.1. Centered forms

H. Ratschek gives a definition of the centered form based on the Taylor expansion of the object function [23]. Let $f(x) = p(x)/q(x)$ be a rational function of one real variable $x \in X$ and let n be the maximum of the degrees of the polynomials p and q. The centered form of f is defined as

$$F(X) = f(m) + G(X - m),$$

where m is the mid-point of X,

$$G(Y) = \frac{\sum_{i=1}^{n} a_i Y^i/i!}{\sum_{i=0}^{n} q^{(i)}(m) Y^i/i!},$$

and $a_i = p^{(i)}(m) - f(m)q^{(i)}(m)$. This expansion can be derived, if instead of the functions $p(x)$ and $q(x)$, we substitute their their Taylor series at the point m.

The above definition of centered forms can be generalized to higher orders by expanding f to its Taylor series and taking the first k terms:

$$F_k(Y) = f(m) + f'(m) \cdot Y/1!$$
$$+ \cdots + f^{(k-1)}(m) \cdot Y^{k-1}/(k-1)!$$
$$+ \frac{a_k^{(k)} Y^k/k! + \cdots + a_{k+n-1}^{(k)} Y^{k+n-1}/(k+n-1)!}{q(m) + q'(m)Y + \cdots + q^{(n)}(m)Y^n/n!},$$

where

$$a_i^{(k)} = p^{(i)}(m) - \sum_{j=0}^{k-1} \binom{i}{j} f^{(j)}(m) q^{(i-j)}(m).$$

III. Results

In this work, we combine three promising global optimization methods:

- interval arithmetic methods,
- experimental design, and
- genetic algorithms,

into a hybrid algorithm.In this algorithm:

- Interval arithmetics gives us simple methods to estimate the range of functions to be optimized.
- Experimental design and factorial analysis helps to find the essential parameter values while
- Genetic algorithms adaptively guides the search.

We have applied this algorithm to robot control and to other engineering problems. The results of this application are much better than for other known methods.

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